Understanding the Remainder When Dividing by Fractions

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Abstract:

Whole number division is extensively modeled to help children intuitively make sense of the remainder as a fractional part of the divisor. To conceptualize division by fractions, children also need extensive modeling of these problems embedded in real-world context. Instruction should explicitly combine the paper-and-pencil algorithm with the three models of area measurement, dimensional analysis, and repeated subtraction. In this way, teachers can better help children connect their understanding of the remainder when dividing by fractions to their understanding of remainder in whole number division.

Understanding the Remainder When Dividing By Fractions

Given a division problem with whole numbers, most students can easily describe what the remainder means. However, when given a division problem in which the divisor is a fraction or mixed number, attaching meaning to the remainder is not so easy. In fact, many children (and adults) who correctly perform the paper-and-pencil calculation, will incorrectly describe what the remainder means. Embedding division by fractions and mixed numbers into real-world measurement problems can be very helpful to learners who are struggling to make sense of these calculations.

Example 1: $23 \div 7 = 3$, remainder 2

Children who have developed a part-whole concept of division, when asked what the remainder means, will reply that there are 2 “things” left over. Or, they will say that the remainder 2 means that each person will receive 3 wholes and $\frac{2}{7}$ of whatever you are dividing up and they will tend to name the things as cookies, cakes, etc. These answers indicate that, even for problems presented without context, children tend to use context when explaining remainders.

Division by fractions and mixed numbers is far more complicated. Even children who understand the paper-and-pencil algorithms, very often cannot meaningfully describe what an existing remainder might mean.

Example 2: Perform the calculation and describe what the remainder means.

$$5 \div 1\frac{1}{2} =$$

$$\frac{5}{1} \div \frac{3}{2} =$$

The answer is 3 and $\frac{1}{3}$ is left over.

$$\frac{5}{1} \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$$

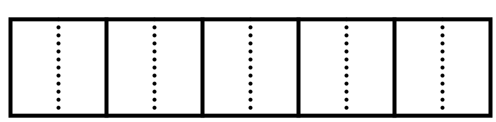
Ask children, "What does the one third mean?" If they have an answer, children will typically say one third of a pie or cookie or whatever you are dividing up. This answer, although obtained from a correct calculation, indicates an incorrect interpretation of the remainder.

Example 3: Darnisha had 5 yards of fabric to make cheerleading flags. Each flag required $1\frac{1}{2}$ yards of fabric. How many flags could Darnisha make? How much fabric was left over after the flags were cut out?

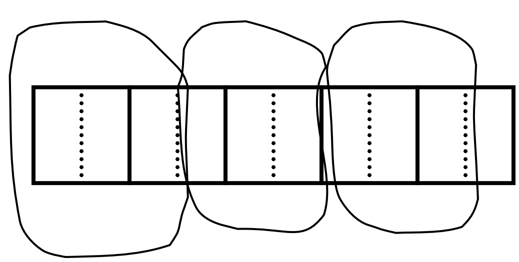
Children will typically answer the first question correctly, but will incorrectly respond that $\frac{1}{3}$ of a yard was left over.

Let's look at a repeated measures model of this problem.

(a) model 5 yards divided into halves



(b) partition pieces of $1\frac{1}{2}$ yd., or $\frac{3}{2}$ yd.



The model illustrates that 3 flags can be made. The remaining piece left over is $\frac{1}{2}$ of a yard.

What then, does the $\frac{1}{3}$ obtained in the paper-and-pencil calculation mean?

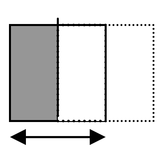
How can we, as teachers, help children conceptually reconcile this answer of 3 flags and $\frac{1}{2}$ of a yard left over with the paper-and-pencil calculation that yields an answer of $3\frac{1}{3}$?

The problems presented to children should be heavily embedded in real-world context. In addition, children should be taught to model these problems in three different ways:

- Area measurement models
- dimensional analysis and,
- repeated subtraction.

Area Measurement Model

Examine a measurement model of the remainder.



1 yard

The shaded area represents both $\frac{1}{2}$ and $\frac{1}{3}$

- $\frac{1}{2}$ of a yard is left over
- $\frac{1}{3}$ of a flag is left over

Dimensional Analysis

$$\frac{5\ yd}{1} \times \frac{1\ flag}{1\frac{1}{2}\ yd} =$$

$$\frac{5\ \cancel{yd}}{1} \times \frac{1\ flag}{\frac{3}{2}\ \cancel{yd}} =$$

$$\frac{5}{1} \times \frac{2\ flags}{3} = \frac{10}{3}\ flags = 3\frac{1}{3}\ flags$$

In dimensional analysis, the units are recorded in both numerator and denominator.

Common units are cancelled in the same way that factors of one are removed.

The unit, flags, indicates that the final answer is given in terms of flags.

Repeated Subtraction

In this model, amounts of $\frac{3}{2}$ yd are repeatedly subtracted until a remainder less than $\frac{3}{2}$ is obtained.

$$\frac{5}{1} \times \frac{2}{2} = \frac{10}{2}$$

Convert 5 yards to $\frac{10}{2}$ yards

$$\begin{array}{r} \frac{10}{2} \\ -\frac{3}{2} \\ \hline \frac{7}{2} \\ -\frac{3}{2} \\ \hline \frac{4}{2} \\ -\frac{3}{2} \\ \hline \frac{1}{2} \end{array}$$

Successive subtractions of $\frac{3}{2}$ yards

3 pieces of length $\frac{3}{2}$ yd. can be cut

$\frac{1}{2}$ yard is left over

Paper-And-Pencil Algorithm

Finally, having made three different representations of the problem, one can see that making sense of the remainder in the paper-and-pencil diagram is a matter of understanding the meaning of the divisor.

$$\frac{5}{1} \div \frac{3}{2} =$$

$$\frac{5}{1} \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$$

The divisor is the **amount needed to make one flag**
$\frac{3}{2}$ yd. = **1 flag**

The answer is $3\frac{1}{3}$ **flags**.

The area measurement model and repeated subtraction help the student see that
$\frac{1}{3}$ *of the divisor* is left over.
$\frac{1}{3}$ of $\frac{3}{2} yd = \frac{1}{2} yd$

Summary

In textbooks, whole number division is extensively modeled to help children intuitively make sense of the remainder as a fractional part of the divisor. To conceptualize division by fractions, children also need extensive modeling of these problems embedded in real-world context. Instruction should explicitly combine the paper-and-pencil algorithm with the three models of area measurement, dimensional analysis, and repeated subtraction. In this way, teachers can better help children connect their understanding of the remainder when dividing by fractions to their understanding of remainder in whole number division.

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