Mastery Learning in Calculus II

Ryan S. Gantner
Saint John Fisher College

Abstract: This article presents a discussion of the benefits and drawbacks of mastery learning used in the author's calculus II class. The traditional mastery learning structures have been adapted to a college setting.

In the 1960s, the ideas of mastery learning were sown by Bloom (1968) and Keller (1968). Bloom's Learning for Mastery (LFM) method became a popular source of experiment, research, and critique. Bloom suggested that students who fail to succeed on an initial formative assessment of a particular topic should work with their cohorts in order to gain mastery of the topic. The students would then be re-assessed, and the class would not proceed to the next topic until the entire class had mastered the one at hand. Keller's Personalized System of Instruction (PSI) was a more individualized form of a similar concept: students work to gain mastery of a particular topic before moving to the next topic. Although the definitions of LFM and PSI appear to have evolved slightly in the literature over time, the basic idea of mastery learning can be summarized in the mantra that, when given enough time and proper motivation, everyone can learn.

This paper presents a discussion of the effects of two Calculus II courses whose structures were similar to the LFM and PSI schemes, but with notable differences. For two consecutive semesters, a Calculus II course at Saint John Fisher College was taught using the techniques described in this paper. No scientific studies of the advantages or disadvantages of the method were conducted. Rather, this paper presents a thorough discussion of the advantages, drawbacks, headaches, and joys related to mastery learning as perceived by me as the instructor.

Background and Purpose

The curriculum of a second semester calculus course at most colleges and universities in the U.S. often contains a hodgepodge of integration techniques and a bag of tricks for determining the convergence of series. The course naturally breaks down into a collection of many small topics which are brought together into a few unifying themes: integration of functions, convergence of series, and power series representations of functions. It is easy to see how a student might become overwhelmed by the sheer quantity of techniques bearing fancy calculus names.

In order to help the students easier digest the contents of such a course, I employed some aspects of mastery learning in the grading structure of two Calculus II courses during the 2007-2008 school year. Since the 1960s, many studies have been done regarding the educational ramifications of such a grading structure. Not only do these studies reach a number of (sometimes contradictory) conclusions regarding the effectiveness of mastery learning, the very definition of "mastery learning" (or "mastery grading") also seems to evolve from one paper to another. Furthermore, to my knowledge none of the situations described the literature reflect the particular structure used in these courses.

This paper has several purposes. One purpose is to simply describe the method used in the Calculus II classes. After establishing the methods and structure, the motivations for such policies will be given and the goals explicitly stated. There is a reflection on how well these goals were met. Finally, some issues which arose in the courses are discussed.

Course structure

The setting is a masters-granting institution which retains its liberal arts core in the undergraduate curriculum. In the fall 2007 section of Calculus II there

were 28 students, and there were 22 students in the spring 2008 section. A handful of students in each of these sections will become math majors, primarily pursuing careers in secondary education. The majority of the students in the class are majoring in biology or chemistry.

Let's give the basic ideas behind the mastery learning structure as they were implemented. The grades for the calculus courses were determined primarily by an elaborate method involving 48 topics. The topics were a set of concepts of which they could gain "mastery". Each topic had four components: a title, a section in the text to read, a set of problems to do for "practice", and an assessment. They gained mastery by passing an assessment which was unique to each topic. For example, topic 1 was on derivatives (which is actually content from Calculus I). In order to gain mastery of topic 1, the students were required to pass a quiz on derivatives. For other topics, the students turned in homework in order to gain mastery. On each assessment, students were given explicit instructions as to the requirements to gain mastery. When an assessment for a topic was submitted for grading, one of three grades was given: E (for Exemplary), S (for Satisfactory), or N (for Not Satisfactory). Grades of E and S were considered to be acceptable to have mastery of a particular topic. If a student did not achieve an acceptable grade, he or she had

the opportunity to re-do the assessment. In the case of homework, the homework was graded and returned to the student, whence they could re-do the necessary problems and resubmit the homework. For quizzes, students who did not pass a particular quiz were given several opportunities to retake a similar quiz to gain mastery. If the student was not able to pass a quiz after a few tries, he or she conferred with me and an alternative means of assessment -- usually an involved homework assignment -- was developed. For any topic oral communication could replace the standard assessment method. I assured the students that if they could have a conversation with me in office hours which convinced me that they had mastered the topic, that I would give them credit for doing so. However, I warned the students that I would have high standards in this respect, so as to encourage the students to work on the problems.

There were a few deadlines built into the topic mastery chart. A student gained mastery of these “deadline topics” simply by attempting the assessments for certain other topics in a timely manner, which was usually two weeks after it was covered in class. The emphasis in the last sentence is on the word “attempt”; students were not required to master the topic at that time, but rather begin their attempt at mastery. Thus, the claim was made that there were

"(almost) no due dates". The only rigid due date was that everything needed to be done by the end of the semester.

The manner in which grades were calculated varied between the fall 2007 and spring 2008 semester. In both semesters, the largest contributor to a student's grade was the topic mastery grade. The other contributors were two semester exams, a final exam, and a skills assessment mandated by the department. In both semesters, the topic mastery grade was determined by a function of the grades on the individual topics. In fall 2007, the topic mastery grade was determined in a "first gap" fashion. The 48 topics were ordered. The number of topics mastered which came before (in the ordering) the first unmastered topic was recorded. That number was divided by 4 to get a score out of 12 (recall there were 48 topics). Eleven points or higher was considered an A, 10-11 an A-, etc. Because of the first gap grading, great care was exercised in determining the order in which the topics appeared. Topics were arranged in an order which combined chronology (topics discussed later in the semester were placed later on the list) and perceived difficulty (topics which I determined to be more difficult were placed later on the list). Students received a bonus if a certain number of topics were given grades of E as opposed to S.

In spring 2008, I abandoned the first gap grading method, for reasons which will be discussed below. At end of the semester, two points were given for each topic mastered with a grade of E and one point for a grade of S. The total number of points was divided by 8 in order to obtain a score out of twelve, similar to the fall 2007 method. Additionally, a fairly complicated categorization of the topics was made in order to ensure that students completed the more basic topics before the more in-depth ones. For instance, a group of topics pertaining to the fundamental theorem of calculus were mandatory to complete before a student could gain credit for any other topic in the course.

Goals of the mastery learning structure

I had several goals for the classes which directly pertained to the mastery learning structure. Some of them were:

1. By being able to turn in assessments multiple times, students will be able to learn from their mistakes, and will not be allowed to be

complacent with their learning. Students cannot “punt” on an assignment which they perceive as being unusually difficult.

2. The lack of formal deadlines will make the course more flexible for the students, allowing them to work around their schedules.
3. The first gap grading (fall 2007) and the categorical divisions (spring 2008) will force students to learn the basics enabling them to better understand the more complicated material.
4. The grading scheme will give the students the (correct) impression that they are not competing against each other, which will encourage collaboration among students.
5. Students will be able work at their own pace, allowing extra time to absorb material for “late bloomers”.
6. Dissecting the course into small, manageable chunks and giving explicit descriptions of what is expected will give the students a sense of accomplishment and allow them to set reasonable goals for focused study.

The goals were achieved with varying amounts of success. Let’s examine each of the six goals listed above.

Goal 1: Students learn from their mistakes. This goal was at least partially met. While the students were forced to fix their mistakes, it was evident that some students benefited from this more than others in terms of their learning. Some students responded well to the written feedback offered on their homework assignments and used the feedback to carefully correct their mistakes. Others, however, simply attempted the problems many times, hoping eventually they would stumble across the correct answer or perhaps simply wear me down enough that I would give them credit. Students became keenly aware of what they didn't know. Overall, this goal was achieved more than in my traditionally-structured classes.

Goal 2: Flexible scheduling. When asked on anonymous surveys, the students universally liked the idea that they could arrange the workload in this class around their schedules. However, it seems that what the students desire is not always best for their learning. No matter what, students will wait until the last minute. Even with some soft deadlines in the spring 2008 version where a student actually bettered his/her grade by attempting assignments on time, the students were still inclined to wait until the last minute, or even miss the deadline. The most common suggestion I received from students on classroom surveys is that I should implement more deadlines. These suggestions astound

me, because the spring 2008 class had trouble meeting the few deadlines they had! In the fall 2007 class, the students turned in a tremendous amount of homework during the last week of class. In spring 2008, the students primarily worked on online homework problems and on the project during the last week of class. Again, many spring 2008 students simply ignored the deadlines even though they understood the negative impact on their grades. Overall, the goal as stated was met. However, it became clearer to me as the year progressed that perhaps this shouldn't be a goal of mine when teaching, because it will encourage bad study habits.

Goal 3: Emphasis on the basics. I took advantage of the first gap feature of grading by inserting some very conceptual materials near the beginning of the course. It was evident that the students would rather forget about their mistakes on this type of assessment and move on (preferring the more formulaic integration techniques), but they were not allowed to do that. I do think that the course structure forced students to think more about the basics and the conceptual material (and populate my office hours!). Whether this resulted in any more thorough understanding of the subject was not immediately clear and was not quantified by any experiment.

Goals 4 – 6. These goals formed the basis for the culture of the classroom. Students worked very hard at the assignments. With the exception of a few spring 2008 students, the classes as a whole put forth a tremendous amount of effort on homework questions. I am not convinced that the amount of effort put forth would have been nearly as great under a more traditionally graded class. Students, particularly in the spring 2008 class, were evidently working together on their homework for the majority of the assignments. In both classes, the students got to know their colleagues very well by the middle of the term, which fostered a healthy learning environment.

Other observations

After teaching the classes, I have made several other observations about the effects of the mastery learning environment on students' learning and my own teaching. Some of them are listed below.

Observation 1. Grading for the fall 2007 class took too much of my time. Even though the homework assignments were given grades that were not detailed (only E, S, and N) and even though I did not feel that I needed to give overly-detailed feedback (because they could re-do the assignment), I still

spent too much time grading homework, particularly at the end of the semester. I found myself trying to get through the homework too quickly and not spending enough time really looking at the student work. In short, I had too much work to do well. So, in spring 2008 I changed much of the homework to an online grading system. The students turned in the majority of their homework online, and the computer graded the problems. The program certainly did handle the initial problem: I was able to free up my time and do a better job grading the assignments that were done by hand. There were, of course, additional drawbacks to the computer grading. The most prominent drawback was that I was not able to witness the student work, which left me more in the dark about the students' performances. Ideally, I would have liked to have spent more time addressing the specific needs of the students.

Observation 2. My grading standards tended to change near the end of the semester. This was more of a problem during fall 2007 than spring 2008. As I was grading the (large) last batch of assignments to come in during the last week of the fall 2007 semester, I was hesitant to give too many unsatisfactory grades due to the first gap grading policy. For example, if a student submitted an assessment for topic 10 which I would have not normally deemed acceptable, yet they have satisfactorily completed topics 11 – 35, then I would

ask myself, “Do I really want to have this one assignment affect the student’s grade so drastically?” More often than not, the answer to that question was “no”, and I found myself being too lenient on some of the last-day submissions. This issue was addressed in spring 2008 by retooling the first gap grading policy and replacing it with a more categorical scheme. When combined with the computer’s non-subjective grading algorithm, the problem of shifting standards was nearly eliminated.

Observation 3. While the lack of deadlines was touted as a feature of this course, it had a negative impact on the amount of feedback I could give to the class as a whole. In a traditionally structured class, when a homework assignment is collected and it is found that a particular type of mistake is common, I can address the mistake in front of the class when handing back the homework. Because the homework assignments in the topic mastery classes came in throughout the semester, and often well after the topics were presented in class, it was difficult to get a feeling for the common mistakes. Even if I could identify common mistakes, it was often so long after the topic was presented that going back over the mistake in class was awkward and confusing. This is another argument for less flexible deadlines.

Observation 4. Perhaps the most important observation deals with student motivation. The students seemed to be extremely motivated by the mastery learning structure. They saw the topic mastery list as a way for them to be personally in charge of their learning and their grades. As many of us know, when a student receives a bad grade on an exam, they tend to blame the exam and not their own deficiencies. Students often build an "it's not my fault, it's the teacher's" attitude toward their grade. The mastery learning structure took away much of this attitude; the students could directly measure their progress and take charge of their education.

Responses to common criticisms of Mastery learning

Several articles have been published with criticisms of the mastery learning paradigm. In this section, some of the aspects of mastery learning which are commonly criticized are examined in the context of the calculus courses.

Criticism (addressed in Block & Burns, 1976): Mastery learning concentrates too much on a procedural/surface understanding of concepts.

There are really two issues embedded in this criticism. First, does mastery learning encourage/foster/develop simply a procedural or surface

understanding of mathematical techniques rather than a conceptual understanding? In the context of this course, I would have to argue that it does not. One of the main components of the topic mastery design in the calculus courses was a project in which students had to model situations using their calculus knowledge. The instructions were intentionally vague so that the student needed to form their own questions about the project. The answers in the project were "messy", which sometimes allows for distinction between students who can see beyond the numbers in what they are doing and those trying to mimic procedures done in class. Besides the project, there were several other topics which could not be mastered without a more conceptual understanding of the mathematics.

The second issue in this critique is: does mastery learning give the *impression* to students that a procedural or surface understanding is all that is important? To this, I must admit that this structure may be guilty. Schoenfeld (1988) noticed many cues that teachers typically give which indicate to the students that they must simply memorize certain facts and spill them on the tests in order to succeed. Teachers are often guilty of this without realizing that they are giving these cues. A teacher can tell the students 20 times that a conceptual understanding is important, but by giving a student a checklist of 48

topics to check off, the student sees that the best way to get a good grade in the course is to complete these items as quickly, painlessly, and superficially as possible. This is a definite drawback to this course structure.

Criticism (Mayer, 1998 and others): Mastery learners are not able to transfer their knowledge to situations which are not introduced in the same format as the topic strand or classroom textbook.

First, as mentioned earlier, there were several topics, including a project, which specifically dealt with the transfer of knowledge to various situations. Thus, this issue was specifically addressed in the materials. Secondly, even if the students did not understand how to transfer their knowledge to different situations, I feel that this is no worse than the typical Calculus II student. By its nature, Calculus II is rather compartmentalized and students have a difficult time relating many of the concepts to non-classroom situations, despite the best efforts of textbook writers and teachers.

Criticism (addressed in Gentile & Lalley, 2003): Mastery learning encourages students to forget acquired knowledge after they have passed the assessment.

Indeed this may have happened in my class. One item that was used as motivation against this is the cumulative final exam. Even so, this criticism is the impetus for some of the changes I would make before doing this again.

Criticism (Slavin, 1987): Mastery learning classes move too slowly and cannot complete the material needed in a semester.

Because my class was not structured in the LFM or PSI models of instruction, this criticism of mastery learning does not apply to my class. As the instructor, I set the pace of the course. Indeed there was plenty of time in order to address the necessary content of the calculus II curriculum.

Criticism (Slavin, 1987): Mastery learning takes too much of the instructor's time while garnering only modest benefits.

The fall 2007 calculus class did take too much of my time, as mentioned earlier. However, this was largely remedied in the spring 2008 class due to changes in the way homework was submitted. In my opinion, the extra motivation of the students greatly outweighs the cost of some extra time on my part.

Conclusions: One note that may have been buried in the details of this paper is that the students in my mastery classes genuinely liked the approach to grading and were far more motivated to do the work than any other set of students I've had in the past. By this aspect alone, I am easily willing to consider the mastery grading setup of these calculus classes to be a success.

If I were to do this again, I would certainly make some changes. The biggest changes come from the criticism that students will view the topics as simply a checklist of things to do and then forget. Perhaps fewer topics, emphasizing depth, would be more applicable. Also, in order to encourage retention and further study, a set of enrichment projects (Gentile & Lalley, 2003) or a more threatening summative evaluation at the end of the term may be in order.

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Biographical Sketch: Ryan Gantner is an assistant professor in the department of Mathematical and Computing Sciences at Saint John Fisher College in Rochester, NY. He has a Ph.D. from the University of Minnesota and works primarily in the area of probability.
rgantner@sjfc.edu