Preservice Point of View: Verbal Interpretations of a Division Problem

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This section of the *LATM Journal* is designed to link teachers and future teachers. In each journal, responses to a mathematical task by preservice teachers are presented. It is anticipated that these responses will provide insight into understanding, reveal possible misconceptions, and suggest implications for improved instruction. In addition, it is expected that this section will initiate a dialogue on concept development that will better prepare future teachers and reinforce the practices of current teachers.

Preservice elementary education students enrolled in a junior-level mathematics course specifically designed for elementary education majors were given the following arithmetic problem: $18 \div 6$. The students were asked to write two "word problems" that would result in having to simplify the given arithmetic problem.

When studying the operation of division, students typically are taught through use of a "fair share" model and a "grouping" model. In the "fair share" model, a known amount is shared equally among a known number of people, entities, containers, etc. What is not known is how many of the known amount should be shared with each person, entity, or container. An example of a "fair share" problem would be as follows:

> Ms. Ima Teacher has 6 students in her after school tutoring group. She bought a box containing 18 colored pencils. How many pencils would each student get if they shared the pencils equally? *Answer: Each student would receive 3 pencils if 18 pencils were shared equally among 6 students.*

In the "grouping" model, the unknown is how many groups of a specific size can be made from a known quantity. An example of a "grouping" problem would be as follows:

> Ms. Ima Teacher has 18 pencils. She wants each of her students to have 6 pencils. How many students could have 6 pencils if Ms. Teacher only has 18 pencils? *Answer: Three (3) students could receive 6 pencils because there are 3 groups of size 6 in 18.*

Thirty-eight preservice teachers participated, but not all provided two problems. There were 73 responses. Forty-six problems involved the "fair share" method. Twenty-one problems involved the "grouping" method. Two of the participants stated the problem in words, "What is the quotient when 18 is divided by 6?" Two of the problems did not relate correctly to the arithmetic problem. One problem appeared to have been intended as a "sharing" problem, but was not stated clearly. One problem involved the formula for the area of a rectangle, which is not directly a "grouping" problem but more information is provided within the problem section below.

Forty-six of the 73 responses involved the "fair share" method, which is approximately 63 percent. Of the 34 students who provided two correct types of problems, fifteen wrote two "fair share" problems, four wrote two "grouping" problems, and fifteen wrote one of each type. The fact that nearly two-thirds of the students wrote problems demonstrating the "fair share" method is worthy of note in itself. Is there something particular in the manner that division is taught that leads to this interpretation? It would be interesting to conduct a study with a larger sample size to see if this preference of the "fair share" method holds in general.

Read over the sample problems below and one can see that some are more "sophisticated" than others. In looking at the incorrect problems, teachers of preservice teachers may use these errors to facilitate a discussion of misconceptions. In fact, the authors suggest an exercise of this type as a critical thinking activity. Have the students write problems relating to some operation and have the students critique the "correctness" of the problems. Note that it is

suggested conducting an exercise of this type in a classroom where professionalism and trust are already established and students read each others' problems anonymously.

"Fair Share" Samples

1. The farmer has 18 sheep that he wants to place into 6 pens. The farmer wants the same amount of sheep in each pen. How many sheep would be in each pen?

2. Sara has 18 stickers that she wants to share with her 6 friends. How many stickers would each friend receive if Sara gives each friend the same number of stickers?

3. Betsy wants to have a garage sale. She has 6 boxes and 18 toys. If she wants to put the toys evenly into each box, how many toys would be in each box?

Note: Better wording here would be "If she wants to put an equal number of toys in each box…" but the intention is understood.

4. Ruth baked eighteen cookies. She wants to share them with five of her friends. If she and each of her friends receive the same number of cookies, how many will each have?

"Grouping" Samples

1. Sally spent eighteen dollars on boxes of cereal. There is no tax on the cereal. If each box costs six dollars, how many boxes did she buy?
2. Eighteen students are out on the playground. For a game, these students need to be split into teams of six. How many teams can the students be split into?

3. Lisa's flower shop is selling half dozens of roses. If she has eighteen roses, how many half dozen sales can she make?

4. I have 18 pieces of candy to put into groups of six. How many groups of six will I have?

Incorrect Samples

1. There are 18 teams in the soccer tournament. If there is only enough time for 6 games in the first round, how many teams will actually get to play? Show all work.

Note: Here is a classic example of using the appropriate numbers, but the numbers here should not be divided. This problem can be answered, but not with division. If there is time for 6 games, then 6x2=12 teams will actually get to play.

2. There are 18 children and 6 pies. If you wanted to distribute the pie evenly, what fraction of a pie would each child get?

Note: Here is another classic example; however division is involved in this problem, but not in the order as requested. This problem results in the arithmetic problem 6÷18= 1/3 indicating that each child would get 1/3 of a pie.

3. Tony made cookies. The recipe made 18. If 6 friends of his eat all of the cookies before he gets any, how many did they each eat?

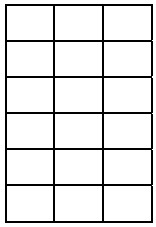
Note: Here is the problem in which the student probably intended it to be a "fair share" but left out the fact that each of Tony's friends ate an equal amount of cookies.

Rectangle Problem

The area of a rectangle is 18". If we know one side is 6", what is the length of the other side?

Note: At first examination, this example does not seem to relate to either category of "fair share" or "grouping" models. However, frequently, a rectangular array is used when the topic of "area" is first introduced in mathematics. At that point, one realizes there could easily be a foundation for the response using the fair share and grouping models.

When one side of a rectangle is 6 units, the array could be drawn as:



Explanations may then state that there are 6 rows of 3 items each and "6 groups of 3."
Other related statements can clearly be categorized into the two types of models.
"Fair share" model - If there are 6 rows, how many will be in each row?
"Grouping" model - If a column has a measure of 6 units, how many columns will there be?