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The LATM Journal is a refereed publication of the Louisiana Association of Teachers of Mathematics (LATM). LATM is an affiliate of the National Council of Teachers of Mathematics (NCTM). The purpose of the journal is to provide an appropriate vehicle for the communication of mathematics teaching and learning in Louisiana. Through the LATM Journal, Louisiana teachers of mathematics – and all teachers of mathematics – may share their mathematical knowledge, creativity, caring, and leadership.

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Euclidean Set Bisectors

Scott Beslin
Nicholls State University

Abstract:
The author examines one alternative interpretation of bisector *from elementary geometry. A guided discovery approach is used to formulate definitions and make conclusions about set bisectors according to this interpretation.*

The purpose of this article is to explore one alternative definition of *bisector* from geometry. The article may be of interest to middle school, high school, and college geometry teachers and their students. In particular, teachers may find it useful in the design of a somewhat "open-ended" project for guided self-discovery or collaborative learning. The article is written in a guided discovery approach. The author welcomes feedback related to teachers' classroom experiences, and their students' investigations of teacher-formulated or student-formulated definitions of *bisector*.

Given two distinct points A and B in the (Euclidean) plane E, certainly the midpoint M of segment AB satisfies AM = BM. In other words, a point M can be found which is equidistant from A and B. Then it is true that the set $\{X \in E : AX = BX\}$ is not empty. With a moment's reflection, one can determine other such points. Let L be the line passing through M and perpendicular to segment AB. If Y is any point on L, and $X \neq M$, then triangle AYB is an isosceles triangle with base segment AB; said differently, $Y \in \{X \in E : AX = BX\}$. Conversely, if Y is any point equidistant from A and B, then Y must be on L. The set $\{X \in E : AX = BX\}$, then, is precisely the line L. Geometry students know L as the "perpendicular bisector" of segment AB. Note that L depends only on the given points A and B. Consequently, one can place only the points A and B in a set S and say that the bisector of S is the line L. Here is the symbolic notation.

$$S = \{A, B\}.$$

By definition, bisector of $S = \{X \in E : AX = BX\}$.

From geometry, L = bisector of S.

Note that S is a set of points, and its bisector is also a set of points. Let's "name" the bisector set of S and write bis(S). When S is a set consisting of two distinct points A and B, bis(S) is the line L described above. Do not confuse the set S with its bisector. For example, if bis(S) = L is the line given above, it need not be true that S = {A, B}. In fact, any set of two points on line AB which have midpoint M also have bisector L. There are other pairs of points as well.

The idea now is to speak of bisectors of sets S which do not necessarily consist of just two points. Let's write definitions and observations based on the work and deductions above.

(1) S is any set of points in the Euclidean plane *E*. The set S may be empty.

(2) (*Definition*) The bisector set of S, bis(S), is the set of points in *E* defined by: bis(S) = {X $\in$ *E* : PX = QX for *all* points P and Q in S}. In other words, bis(S) is the set of all points in the plane equidistant from *every* point in S.

(3) Suppose S is given and bis(S) = T. Consider the open sentence: bis(□) = T, with the "( )" to be filled in. Then there may be possibilities other than S itself. In other words, the set function "bis" is not one-to-one.

At this point, let's consider the case of bis(K), where K is a line. If A is any point on K, then the distance from A to A is 0, and no other point B (≠ A) of K satisfies AB = 0. Therefore no point of K is in bis(K). Suppose B is a point not on K. Then there is a point H of K so that BH is the shortest distance from B to K; the point H is the "foot" of the perpendicular segment "drawn" from B to L. No other point of K has this relationship with B. Consequently B is not in bis(K). Therefore neither a point on K nor one off K is in bis(K). The conclusion is that bis(K) = ϕ, the empty set.

Remember the set S ={A, B} of two distinct points from the opening paragraph. The conclusion was that bis(S) = L, the line perpendicular to segment AB and passing through its midpoint. Observe now that bis(L) = ϕ. It is therefore not necessarily true that bis(bis(S)) is S.

Many “operations” and functions in mathematics have the property that two applications of them return the original input. For example, if x is a nonzero real number, then the reciprocal of the reciprocal of x is again x. This specific example, $f(x) = \frac{1}{x}$, $x \neq 0$, is an example of a function which is its own inverse function. Algebra gives many more. Note that the bisector set function does not work this way. Let’s list this fact as another observation.

(4) For a set S, it is not necessarily true that bis(bis(S)) = S.

Let’s now find bisectors for other sets S.

What about if S = *E*, the entire plane? If a point X is in bis(*E*), it is equidistant from *every* point in *E*; in particular if K is any line in *E*, the point X is equidistant from every point of K. But it is known that bis(K) = ϕ. Therefore there is no choice but that bis(*E*) = ϕ. Note the deeper result here.

(5) If S is a subset of T, then bis(T) is a subset of bis(S). In particular, if S is a subset of T, and bis(S) = ϕ, then bis(T) = ϕ.

If a point X is *not* in the bisector of S, then by definition, there are two points A and B in S satisfying AX ≠ BX. If the set S itself is empty, no such points A and B can be found (since S doesn’t have any points). Conclusion?

(6) bis(ϕ) = *E* and bis(*E*) = ϕ.

You, the reader, should try one. Suppose S consists of a single point A. Read the definition of bis({A}) carefully. Can you find a point in this bisector? two points? three points? Use the result in (5). Let B be *any* point in *E* other than A. Then {A} is a subset of {A, B}. Therefore bis({A, B}), a line, is a subset of bis({A}). Since B is “arbitrary,” “many”(!) lines are contained in bis({A}).

(7) The bisector of a set with only one point is the plane *E*.

From earlier discussion:

(8) The bisector (either traditionally or by Definition 2) of a set with exactly two distinct points A and B is the line perpendicular to segment AB and passing through its midpoint.

For sets with more than two points, items (5) and (8) above are useful. Let's take a set S = {A, B, C} with three distinct points. Since {A, B} is a subset of S, bis(S) must be a subset of the linear perpendicular bisector of segment AB. Likewise, bis(S) must be a subset of the linear perpendicular bisector of segment BC. Thus bis(S) is a subset of the intersection of these two lines, which is either empty or has exactly one point. In the case of one point, the perpendicular bisector of segment AC also passes through that point because {B} is the intersection of {A, B} and {B, C}. Can you deduce when bis(S) is empty and when it is not?

(9) Let S = {A, B, C} have three distinct points. If A, B, and C are collinear, then bis(S) = ϕ. Otherwise, the points A, B, and C lie on a circle and bis(S) is the center of that circle.

Let's use the word "co-circular" to describe "lying on the same circle."

Slightly modify the preceding argument for sets S with more than three points.

(10) Let S be a set with more than three points. Then there are two possibilities: the points of S are co-circular, in which case bis(S) is the center of that circle; or there is no circle containing all the points of S, in which case bis(S) = ϕ.

Here's a summary of some of the main results of set bisector exploration.

(11) If S is any subset of the plane *E*, then bis(S) is one of the following: *E*, ϕ, a line, or a set with exactly one point. Moreover, bis(bis(S)) is either ϕ or *E*.

Here are some questions for reflection, and for geometry students.

1. What are the possibilities for the *intersection* of S and bis(S)?
2. Are there sets S for which bis(bis(S)) = S ?

3. Re-examine item (5) and address its converse. Suppose bis(T) is a subset of bis(S). Is there any relationship between S and T ?

4. In each case, fill in the ( ) with two *different* sets of points.

   (a) bis(□) = { (0,0)}, the set consisting of the origin alone

   (b) bis(□) = { (x, y): $y = x$ }, the set of all points on the line whose equation is $y = x$

5. In each case, find bis(S).

   (a) S = { (0,1), (1,3), (-1, -1) }

   (b) S is the set of points in the intersection of the parabola $y = x^2$ and the line $y = x$.

   (c) S is the set of points (x , y) satisfying $x^2 + y^2 = 1$ and lying above the y-axis.

Concluding Remarks

The notion of "bisector" in this article is only one possible alternative to the idea of perpendicular bisector of a line segment. Certainly there are equally appropriate and meaningful alternative definitions of this concept (what comes to mind when "*bisecting* a set" or "*bisecting* a geometric figure" is heard?). Innovative teachers can propose their own definitions and then devise clever explorations for their students. Inquiries may also be based on current definitions extended to three-dimensional space.

The term *bisector* in the article was chosen primarily because of the discovery argument related to two distinct points, presented in the opening paragraphs. This choice for the definition proposed in the article should not be confused with traditional meanings of *bisector* for which it is clear that geometric objects are "divided" into two parts. For example, a circle is said to be *bisected* by a line containing a diameter of the circle.

Acknowledgement: *The author wishes to thank the reviewers of this article for their thoughtful and useful suggestions. One reviewer pointed out that the idea of the bisector of* two sets *of points, defined as the collection of points equidistant from the two sets, is used in computational geometry and is expressed in terms of infimum or greatest lower bound. See reference* [3] *for more information.*

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[1] E.B. Burger, D.J. Chard, E.J. Hall, ***Geometry***, Holt Mathematics Textbook, Holt McDougal, 2007.

[2] M. Serra, ***Discovering Geometry: An Investigative Approach***, Key Curriculum Press, 2002.

For advanced reading:

[3] Martin Peternell, *Geometric Properties of Bisector Surfaces*, **Graphical Models**, Vol. 62 (2000), pp. 202-236.

Scott J. Beslin *is professor of mathematics at Nicholls State University. He has been an active, successful teacher and scholar in his 25 years in the field. Beslin's primary research area is algebra, but he has contributed papers and made presentations in linear algebra, number theory, analysis, calculus, geometry, topology, and trigonometry. He has also directed many graduate and undergraduate research projects and enjoys working with students. Beslin may be contacted at scott.beslin@nicholls.edu.*

Building a Mathematical Foundation: Scavenger Hunting and Other Engaging Activities

Faye Bruun
Texas A&M University – Corpus Christi

Abstract:
A former middle school teacher designs a scavenger hunt for low income students visiting a university campus at a summer mathematics camp. The goal of the camp is to provide an engaging and educational experience in mathematics by having the students use measurement tools and geometric formulas after hunting for university buildings.

Texas A&M Corpus Christi invited middle school students from six South Texas school districts to the campus for engaging mathematical activities focusing on NCTM (2000) Geometry and Measurement Standards. By students participating with a scavenger hunt, they **a**nalyzed characteristics and properties of three-dimensional geometric shapes of buildings and developed mathematical arguments about geometric relationships (the Geometry strand). Students distinguished between the measurable attributes of objects and the units, systems, and processes of measurement by using tools such as a trundle wheel and clinometer to measure distances and heights (the Measurement strand).

The goal of the camp was to provide an engaging and educational experience for the students in mathematics. GEAR UP is an acronym for Gaining Early Awareness and Readiness for Undergraduate Programs, a grant program designed to increase the number of low-income students who are prepared to enter and succeed in postsecondary education. These middle school students were accompanied by their regular math teachers. The teachers attended workshops in the spring to work through the planned activities and to offer modifications that would be helpful to the program with a focus on Geometry and Measurement. Participants of the learning experience also included Faculty Fellows, university professors who worked with the school districts and designed the summer math program. As an Assistant Professor in the College of Education, my role was to design the scavenger hunt.

DAY 1: MORNING

During the first day of the camp, the students went on a walking tour of the campus of Texas A&M Corpus Christi as a preview for the scavenger hunt on the next day. The campus is located on an island and houses many round-shaped buildings. None of the buildings are taller than three stories because of the proximity to the Naval Air Station runway where Navy aviators are trained. Many of the students had never been to a university campus before and were able to see university buildings containing lecture halls, science labs, dining facilities, dormitories, as well as the fieldhouse and tennis courts.

DAY 1: AFTERNOON

The students then spent the first afternoon exploring geometric shapes in order to establish the prior knowledge necessary to complete the geometry and measurement concepts for solving the problems gathered from the Scavenger Hunt. As a focus, the following activity for volume of a cylinder was presented:

> Take two identical sheets of clear transparency paper measuring 8 ½ inches by 11 inches. Roll one sheet into a short cylinder and the other into a tall cylinder taping each cylinder together with masking tape. Set them both on a flat surface. Does one hold more than the other? Have students vote.
>
> Place the taller cylinder inside the shorter one. Fill the taller one with popcorn; then slowly lift it, removing it from the shorter cylinder. Which holds more?

The students were able to observe that the shorter one has a greater volume. This activity was adapted from *Figure This! Math Challenges for Families* from NCTM.

Using Giant Geometric Shapes from ETA © with transparent faces and color bases, students were challenged to estimate the volume of various shapes and place them in order from greatest to least volume. The ten shapes included cone, cylinder, cube, rectangular prism, square pyramid, hexagonal prism, triangular prism, triangular pyramid, and two hemispheres. Through observation and inference, students were able to place the figures in order from greatest volume to least volume. Later, they were encouraged to refer to the shapes for reference as they were reviewed on the formulas for finding the volume of each shape.

The next activity involved using the smaller Folding Geometric Shapes © from ETA that were constructed using the metric system and length of 8 cm. Students were shown the website depicting liquid from a cone being poured into a cylinder tank (http://nlvm.usu.edu/en/nav/frames_asid_275_g_3_t_4.html) to help them discover the relationship between the cone and the cylinder with the same base and height, as well as the pyramids and prisms with the same base and height as having the ratio of 1:3.

Once students and teacher finished the discussion, students mathematically calculated the volume of each shape of the Folding Geometric Shapes to confirm the accuracy of their initial guesses about volume using the Geometric Solids Activity table by starting with perimeter of the sides of the solids. Because of the thickness of the plastic, measurements between students were slightly "off," depending on if they measured from the inside edges or the outside edges. Starting with perimeter, students filled in the charts. This is the first one for perimeter:

Use the outside of the plastic and measure to the nearest cm:

	Draw shape.	Write formula.	**Perimeter of bases in cm:**
Square side of cube			
Rectangular side of rectangular prism			
Circle of cylinder			
Equilateral Triangle			
Regular Hexagon			

Then students did the same for Area of bases in square centimeters and Volume in cubic centimeters for the cube, the square pyramid, the rectangular prism, the cylinder, the cone, the triangular prism and pyramid, and the hexagonal prism. Volume, or the capacity of an object, is sometimes confused with surface area. At first glance, the formulas for finding each appear somewhat similar. A helpful way to compare the two is to explain surface area as the amount of area on the outside of a shape, and the volume as the amount of space inside a shape. After computing both lateral and total surface area in square centimeters, for enrichment students traced nets on cm square paper to compare surface areas to the table.

DAY 1: HOME ACTIVITY

The students were given the following as they boarded their buses to go home:

At Home Assignment

> Tell your family about the tall and short tower. Ask them to guess which holds more. Look in your cupboard and see what items come in differently-shaped or differently-sized rectangular prisms or boxes. Find two differently-shaped containers that hold the same amount. What are the contents of each? Bring them to the university tomorrow.

DAY 2: MORNING

Students discussed their "home assignment" by sharing items they had brought from home. The purpose of the assignment was to provide a family discussion of what we were doing at camp and for the students to share with each other items they brought from home.

The students were put in teams with an accompanying teacher. The students from the six school districts were divided into different groups so that they could meet and interact with students from other schools. Each group was given a trundle wheel, which is used to measure long distances in the metric system. A counter "clicks" each revolution measuring one meter, allowing students to use this real-world tool to accurately track distance covered either in a straight line or the circumference of a round building.

Using the Pythagorean Theorem, a meter stick, and an instrument called a clinometer, students were able to measure heights of buildings. Students pointed the clinometer at the top of a building, pulled the trigger, and waited for the graduated disc to stop spinning. Clinometers show all possible angles, but for this activity, a 45-degree angle was used. Students could tell each other when the angle of 45 degrees was seen. They used the trundle wheel to measure the distance from the base of the building to where they were standing and used the meter stick to find the distance between the height of their eye on the clinometer and the ground. Adding these two distances together gave the approximate building height. Directions to make a simple clinometer are found at the following web site: http://www.nativeaccess.com/ancestral/totems-intro.html. This web site also has a good visual of the process for using the 45-degree angle and distance from the building and height of the ground to the clinometer in order to calculate height of the building.

After reviewing the trundle wheel and clinometer, the students were given the following instructions:

Scavenger Hunt

Rules: Stay with your team. Walk on sidewalks. Use the following clues to find the place and make measurements with a trundle wheel and clinometer.

After locating particular buildings students had seen yesterday utilizing a map, they were asked questions such as:

For a round building: Measure the circumference and the height of the building.

For a campus statue: How many sides does the short wall have where the statue sits?

What is the name of this polygonal shape?

Measure the height of the statue.

Measure the length of the wall of the plaza.

For the water tower: Find the cylinder tank that contains the water used by the university for the heating and cooling of all of the buildings on campus. Measure the circumference and the height.

Find the statue at the entrance of the university. What shape are the five “pointed” structures? Measure the base and the height.

For a fountain plaza: How many sides does the short wall have where the fountain sits?

What is the name of this polygonal shape?

What is the perimeter of this fountain?

For the fieldhouse, the clue was: This building has a gymnasium and racquetball courts. Measure to compute the square footage of the building.

How many tennis courts are there? What shape are they? What is the length and width?

This was the Bonus Question: How long is the Hike and Bike trail in kilometers?

DAY 2: AFTERNOON

After gathering all the measurements from the campus buildings, the students came back to the classroom (and air conditioning) and completed the following:

Scavenger Hunt Activity Sheet

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1. What is the area of the base of the Student Services Center?
2. What is the volume of the Student Services Center?
3. What is the height of the Hector P. Garcia Statue?
4. What is the length of the wall of the Hector P. Garcia Plaza?
5. What is the area of the base of the water tower?
6. What is the lateral surface area of the water tower?
7. What is the total surface area of the water tower?
8. What is the volume of the water tower?
9. What is the area of the base of the "points" on the sign at the entrance?
10. What is the lateral surface area of one of the "points?"
11. What is the total surface area of one of the "points?"
12. What is the volume of one of the "points?"
13. What is the area of the base of the Education Center for Math and Science?
14. What is the volume of the Education Center for Math and Science?
15. What is the length of the wall of the Lee Fountain?
16. What is the square footage of the fieldhouse?
17. What is the area of a tennis court?
18. Bonus: What is the length of the Hike and Bike trail?

CONCLUSION

The two days of the camp were successful in orienting the students to a university campus and to applying geometric formulas to real world buildings. They had the opportunity to use measuring tools and geometric concepts to help build a mathematical foundation in the NCTM Geometry and Measurement Standards. A similar scavenger hunt could be designed for your school campus by locating buildings, walls, and structures for students to measure with tools such as the trundle wheel and clinometer.

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Faye Bruun *is currently an Assistant Professor in the Department of Curriculum & Instruction at Texas A&M Corpus Christi. She teaches graduate courses in mathematics education. Bruun's primary research focus is using literature and manipulatives to teach mathematics conceptually. She is Secretary of Texas Council of Teachers of Mathematics.*

Geometry in Simple Molecular Models

Andy Talmadge
Texas A&M University – Corpus Christi

Abstract:
In this work the geometry of small covalently bonded molecules such as carbon dioxide, boron trifluoride, and methane is explored using the idea of the centroid. Each of the above is described as a particular configuration of points on the unit sphere which has the origin as its centroid.

Introduction

Molecules are electrically neutral groups of two or more atoms held together by covalent bonds (the sharing of electrons). Most common gases and liquids are composed of molecules (air [O_2, N_2, CO_2], water [H_2O], methane [CH_4]). The three-dimensional shape or molecular geometry of a molecule is an important factor in determining the chemical and physical properties of the molecule. Molecular geometries are specified in terms of bond lengths and bond angles. Bond length is defined to be the average distance between the centers of two atoms bonded together. Bond angle is the angle formed between three atoms across at least two bonds.

A number of different models are employed to develop understandings of these shapes. The *ball-and-stick* model of a molecule conceptualizes chemical bonds as a direct link between atoms. The atoms are small balls with holes drilled in them and the bonds are sticks which fit into the holes and link the balls. More generally, the Valence Shell Electron Pair Repulsion (VSEPR) model for covalent bonding of atoms is used to predict the three dimensional shape of molecules. The spatial arrangements of the atoms in CO_2 (carbon dioxide), BF_3 (boron trifluoride), and CH_4 (methane) are explored from a mathematical point of view in what follows.

In each of the cases explored the molecule in question has a central atom (carbon in the cases of carbon dioxide and methane, and boron in boron trifluoride) bonded to some number of identical peripheral atoms. Since the bonded atoms are all the same in each case the VSEPR model predicts the bonds will all have equal lengths. Modeling the atoms as points in three dimensional space, the central atom is assumed to be located at the origin of the Euclidean coordinate system. Each of the peripheral atoms is represented by a point on a sphere of radius 1 centered at the origin.

In this paper, we compute the position of n = 2, 3, 4 points on a unit sphere that has the origin as its geometric center. The positions of these points are shown to correspond to the geometric arrangement of the atoms in the VSEPR models of carbon dioxide, boron trifluoride, and methane.

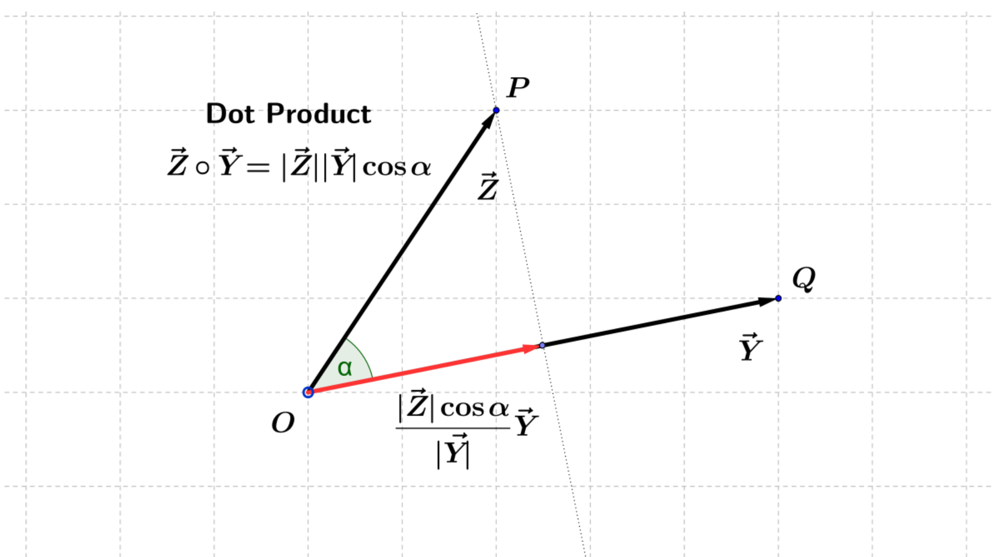
Centroids of Points on a Sphere

Let $P_1(x_1, y_1, z_1), \cdots, P_n(x_n, y_n, z_n)$ be points in Euclidean 3-space. The *centroid* of the points P_i is the point $G_{\{P_1,\ldots,P_n\}}$ whose coordinates are given by the formula:

$$G_{\{P_1,\ldots,P_n\}} = \left(\frac{\sum_1^n x_i}{n}, \frac{\sum_1^n y_i}{n}, \frac{\sum_1^n z_i}{n}\right)$$

Generically speaking, n points will have no three collinear, so that when $n = 3$ the points will form the vertices of a triangle, when $n = 4$ the vertices of a tetrahedron.

The ***dot product*** of two position vectors $\vec{Z} = \overrightarrow{OP} = \langle a, b, c\rangle$ and $\vec{Y} = \overrightarrow{OQ} = \langle e, f, g\rangle$ is the number given by $\vec{Z} \cdot \vec{Y} = ae + bf + cg = |\vec{Z}||\vec{Y}| \cos\alpha$ where α is the angle $\angle POQ$.



Example 1 For $n = 2$, the centroid is the midpoint of the segment connecting P_1 and P_2. If we require the points lie on the sphere of radius 1 about the origin with centroid $G = (0,0,0)$, then $P_1(x, y, z)$ and $P_2(-x, -y, -z)$. Considering the unit vectors $Z_i = \overrightarrow{GP_i}$, then $Z_1 \cdot Z_2 = -1$ and the angle between the vectors is $180°$.

Example 2 For $n = 3$, consider points P_1, P_2, P_3 that lie on the sphere of radius 1 about the origin and that have centroid the origin (i.e. $G(0,0,0)$). The three points cannot all lie on the z-axis so we can choose $P_1(0,0,1)$ and $P_2(x_2, 0, z_2)$, with $x_2 > 0$ (the second point lies off the z-axis – declare this direction to be the positive x-axis and let the y-axis follow from the right-hand-rule) this gives the equations:

$$x_2 + x_3 = 0$$

$$y_3 = 0$$

$$1 + z_2 + z_3 = 0$$

The second equation implies that the three points all lie in the xz-plane. Computing the dot products $Z_1 \cdot Z_j = z_j = \cos\theta_j$ where θ_j is the angle that Z_j makes with the z-axis. The points have coordinates:

$$P_1(0,0,1), P_2(\sin\theta_2, 0, \cos\theta_2), P_3(-\sin\theta_2, 0, \cos\theta_2)$$

where $\cos\theta_2 = -\frac{1}{2}$, hence $\theta_2 = \theta_3 = 120°$. Computing the dot product

$Z_2 \cdot Z_3 = -\sin^2 120° + \cos^2 120° = -\frac{3}{4} + \frac{1}{4} = -\frac{1}{2}$, so that the angle between these vectors is also $120°$. The points are the vertices of an equilateral triangle.

Example 3 For $n = 4$, consider points P_1, P_2, P_3, P_4 that lie on the sphere of radius 1 about the origin and that have centroid the origin (i.e. $G = (0,0,0)$). Without loss of generality we can choose $P_1(0,0,1)$ and $P_2(x_2, 0, z_2)$, with $x_2 > 0$ this gives the equations:

$$x_2 + x_3 + x_4 = 0$$

$$y_3 + y_4 = 0$$

$$1 + z_2 + z_3 + z_4 = 0$$

Computing the dot products $Z_1 \cdot Z_j = z_j = \cos\theta_j$ where θ_j is the angle that Z_j makes with the z-axis. This gives the coordinates of $P_2(\sin\theta_2\,, 0, \cos\theta_2)$. Rewriting the equations above gives:

$$x_3 + x_4 = -\sin\theta_2$$

$$y_3 + y_4 = 0$$

$$\cos\theta_3 + \cos\theta_4 = -(1 + \cos\theta_2)$$

<u>Case 1</u> $(\cos\theta_2 = -1)$ Then $\sin\theta_2 = 0$ and we can conclude that $(x_4, y_4, z_4) = (-x_3, -y_3, -z_3)$. Hence the four points occur in two pairs of antipodal points on the sphere.

<u>Case 2</u> $(\cos\theta_2 > -1)$ If no further constraints are put on points P_3 and P_4 the general equations above locate the midpoint of P_3 and P_4 as $\left(-\frac{\sin\theta_2}{2}, 0, -\frac{1+\cos\theta_2}{2}\right)$ which is clearly antipodal to the midpoint of P_1 and P_2. Moreover, this midpoint is located in the xz-plane. Considering the configuration of points and the definition of centroid, it is clear that pairwise this relationship holds for all pairs of points under consideration.

Squaring the equations and using the fact that the points must lie on the unit sphere yields:

$$2 + 2(x_3x_4 + y_3y_4 + \cos\theta_3\cos\theta_4) = 2 + 2\cos\theta_2$$

or

$$Z_3 \cdot Z_4 = x_3x_4 + y_3y_4 + \cos\theta_3\cos\theta_4 = \cos\theta_2$$

which implies that the angle between Z_3 and Z_4 is the same as the one between Z_1 and Z_2. Since the origin is the centroid for these points introduce parameters α and β with $x_3 = -\alpha\ \sin\theta_2$, $x_4 = -(1 - \alpha)\sin\theta_2$, $y_3 = -\beta\,(1 + \cos\theta_2)$, $y_4 = -(1 - \beta)\,(1 + \cos\theta_2)$. Note that these parameters will insure that the origin will be the centroid. The new parameters are determined by the constraints that $Z_3 \cdot Z_3 = Z_4 \cdot Z_4 = 1$ and $Z_3 \cdot Z_4 = \cos\theta_2$. The unit vector requirements give

$$1 = \alpha^2(\sin\theta_2)^2 + {y_3}^2 + \beta^2(1 + \cos\theta_2)^2$$

$$1 = (1 - \alpha)^2(\sin\theta_2)^2 + {y_3}^2 + (1 - \beta)^2(1 + \cos\theta_2)^2$$

Subtracting one from the other yields:

$$0 = \{\alpha^2 + (1 - \alpha)^2\}(\sin\theta_2)^2 + \{\beta^2 + (1 - \beta)^2\}(1 + \cos\theta_2)^2$$

Since θ_2 can have any value the coefficient terms must be zero, hence $\alpha = \beta = \frac{1}{2}$, making

$${y_3}^2 = \frac{1 - \cos\theta_2}{2}$$

Summarizing what has been found gives:

$$Z_1 = (0,0,1)$$

$$Z_2 = (\sin\theta_2\,, 0, \cos\theta_2)$$

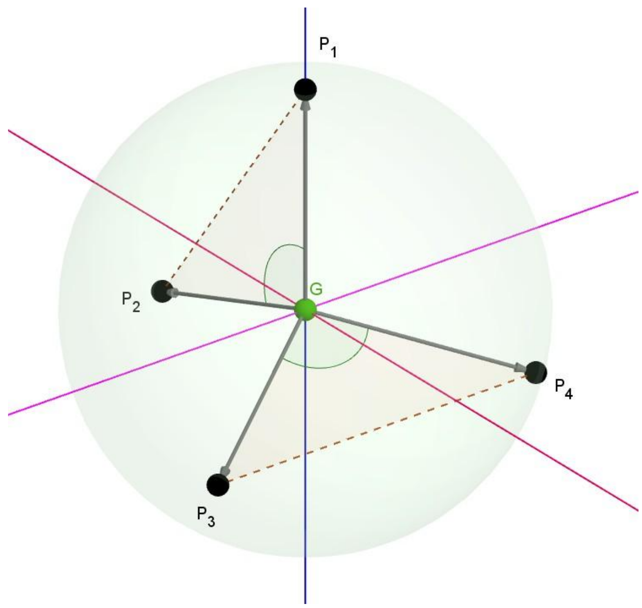
$$Z_3 = \left(-\frac{\sin\theta_2}{2}, \sqrt{\frac{1 - \cos\theta_2}{2}}, -\frac{1 + \cos\theta_2}{2}\right)$$

$$Z_4 = \left(-\frac{\sin\theta_2}{2}, -\sqrt{\frac{1 - \cos\theta_2}{2}}, -\frac{1 + \cos\theta_2}{2}\right)$$

$$Z_1 \cdot Z_2 = \cos\theta_2$$

$$Z_3 \cdot Z_4 = \cos\theta_2$$

$$Z_1 \cdot Z_3 = Z_1 \cdot Z_4 = Z_2 \cdot Z_3 = Z_2 \cdot Z_4 = -\frac{1 + \cos\theta_2}{2}$$



The angle between the first two points (arbitrarily chosen) is the same as the angle between the remaining two points and the angle between each of the initial points and each of the other two points is also equal but not necessarily the same as the first angle chosen. The six angles in question would all be equal when $\cos\theta_2 = -\frac{1}{3}$ or $\theta_2 = \cos^{-1}\left(-\frac{1}{3}\right) \approx 109.47°$. When the six angles are equal the points on the sphere are the vertices of a regular tetrahedron (one of the Platonic solids).

Application to Simple Molecules

In example 1 of the previous section two points on a sphere whose centroid is the origin was only possible if the points were antipodal and hence the points $P_1,\ G,\ P_2$ lie along a line. In the stereochemistry predicted by the VSEPR model, carbon dioxide, CO_2, has this geometry with carbon C at the center of a sphere whose radius is the bond length of the double bonds formed between carbon and oxygen. The bond angle OCO in carbon dioxide is 180°.

In example 2 of the previous section three points on a sphere whose centroid is the origin was only possible if the points were the vertices of an equilateral triangle. Boron trifluoride has a similar geometry with boron at the center of a planar molecule whose shape is that of an equilateral triangle. The bond angle FBF in boron trifluoride is 120°.

In example 3 above four points on a sphere whose centroid is the origin was possible in many ways. Choosing an angle between the vectors that point from the origin to the first two points completely determines the angle between all of the vectors from the origin to the points on the sphere. In one case all of these angles were equal (109.47°); this is precisely the case of the bond angles (HCH) in a methane molecule where carbon is the central atom bonded to four hydrogen atoms.

This work deals with highly symmetric molecules where bond lengths and bond angles are all equal. Molecules with less symmetry, for example, methyl chloride (CH_3Cl), aren't as easily modeled using the ideas presented here.

Conclusion

Many students learn the stereochemistry of covalently bonded molecules in a chemistry classroom and the concept of a centroid in a mathematics classroom. The angles between atoms bonded to a central atom are often memorized in the chemistry classroom. This work provides a mathematical framework that can help students to understand the basis for the angles found in the chemistry classroom, in particular, the mysterious 109.47° angle between the hydrogen atoms in a methane molecule.

Perhaps connecting mathematics (in particular geometry) to problems found in science classrooms will motivate students to explore more mathematical connections. Minimally such connections might open students to ask more questions like: *Why is this true? What mathematics is behind the fact I'm being asked to memorize?*

In the section above methane was found to have geometry similar to a special configuration of four points on a sphere whose centroid was the origin (one where all of the angles between vectors from the origin to these points were equal). A reader might ask if it is possible for there to exist molecules whose geometry arises from more general configurations of these points.

A possible extension of this investigation might be to consider configurations of points where some of the points lie on one sphere and others lie on another sphere. If the requirement that the points have the origin as their centroid is maintained will such configurations give rise to geometries like those found in molecules? In particular, could this give configurations that might arise when bond lengths are not equal?

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Andy Talmadge *has been teaching mathematics and science for 30 years and is interested in the use of models and technology in learning. He directs the Mathematical Sciences Institute held each summer at Tulane University. Andy currently teaches physical science to middle schoolers and mathematical modeling to upper schoolers at Metairie Park Country Day in Metairie, Louisiana.*

Math Circles: Gateways to Mathematical Habits of Mind

Lee Windsperger
Louisiana State University – Baton Rouge

Abstract:
Motivated by the new Common Core State Standards, this article illustrates the Regions of a Circle Problem, a mathematical enrichment activity that is typical for Math Circles nationwide. The Math Circle literature is a valuable source for activities, problems, and projects developed to strengthen students' mathematical habits of mind as required by the Standards for Mathematical Practice.

Math is everywhere! This popular mantra is often supported by math teachers when arguing for the value of mathematics education. Hence good educators stockpile "real world" applications for math skills to serve as evidence of the ubiquity of mathematical concepts and to answer the question: "When will I ever need to use this?" This evidence justifies why almost every degree program and many employers require and value math skills and knowledge for their potential students and employees.

Professor Underwood Dudley gives a different view of why math should be valued in his article *What Is Mathematics For?* (2010). Dudley argues that the idea of specific mathematical content knowledge and skills beyond arithmetic being necessary to prepare the vast majority of students for problems in their future careers is *completely* bogus. He illustrates that though mathematical ideas may be present, the average worker or individual is not required to actually know or do any mathematics. The math has already been done by others or by machines. This does not mean that Dudley thinks that mathematics education is superfluous. On the contrary, he argues that mathematics education is "the best method we have" to teach people reason – a skill that is most certainly valued and relevant in every workplace and aspect of life.

While Dudley's points about the necessity for specific mathematical content knowledge and skills in the average workplace may not be supported by all authors of the Common Core State Standards (CCSS), there seems to be resounding agreement between all parties regarding the importance of mathematics in developing the proper habits of mind for reasoning and problem solving. The CCSS writers' focus on the value of analytical thinking and problem solving is evident when one reviews the *Standards for Mathematical Practice* and their relevance in the development of the proto-type test items for the two new national assessment systems PARCC and SMARTER Balanced Assessment Consortium. Louisiana has selected PARCC assessments to replace all LEAP and *i*LEAP tests as well as end of course tests in mathematics in 2014-15. The *Standards for Mathematical Practice* clearly describe the mathematical habits of mind that math educators are to develop in their students of all levels. These standards state that students:

- make sense of problems and persevere in solving them
- reason abstractly and quantitatively
- construct viable arguments and critique the reasoning of others
- model with mathematics
- use appropriate tools strategically
- attend to precision
- look for and make use of structure
- look for and express regularity in repeated reasoning

To ensure that these standards are being upheld in Louisiana elementary and secondary classrooms, the forthcoming PARCC assessment systems will require from the students a heavy dose of "task-based" problem-solving skills. These tasks can be defined as a problem or set of problems that aim to do more than just focus students' attention on a particular mathematical idea. The tasks provide opportunities to develop or use a particular mathematical habit of mind (Shimizu, 2010). These task-based assessment items are a clear departure from the current assessment approaches to the Grade Level Expectations (GLE's) as implemented through the LEAP exams or college entrance exams such as the ACT or SAT.

The heavy emphasis on the *Standards for Mathematical Practice* and the accompanying high-stakes, task-based assessment items will force many educators to reevaluate their teaching. For their students to be successful in the new tests, teachers will have to find additional ways to address and nurture the mathematical habits of mind in their students. Currently, a standard way such reasoning skills are developed in students is through mathematical enrichment activities in the form of residential summer programs, math contests, and school-based math clubs. These programs' target audience is usually limited to secondary students who have already proved to be successful problem solvers and learners of math concepts and skills. Unfortunately, math phobias, different educational backgrounds, disparate abilities, as well as competition anxiety, can isolate the common student from such enrichment activities. Teachers will be forced to find new methods to create in all students a sense of comfort and direction that enables them to enjoy challenging, open-ended mathematical problems and tasks.

Math Circle literature may prove to be a valuable resource for teachers interested in an inclusive classroom or extracurricular environment to cultivate the proper mathematical habits of mind. The goal of Math Circles is to provide students with a social environment that encourages them to be passionate about mathematics. There are no GLE's to cover, no set curriculum, and no tests. The teacher functions less as a teacher and more as a scribe and moderator for the conversation and exchange of ideas. The students lead the discussion; hence the need to encourage creativity, openness, perseverance, and maturity are paramount. The focus is on doing and exploring mathematics in a fun and informal way, and in the proper setting, the students will naturally discover and exchange new mathematical skills and tools in the context of an engaging problem. This discussion and mathematical journey will only help to foster the participants' mathematical abilities as well as the proper mathematical frame of mind.

The Math Circle movement prides itself on developing, collecting, and refining task-based problem sets and activities for students at all levels. These challenging problems often have captivating and widely accessible starting points and introduce a wealth of deep mathematics. By exploring such problems, the students are able to connect multiple mathematical concepts and are given the opportunity to develop the skills outlined in the *Standards for Mathematical Practice*. There is a growing archive of free, online Math Circle literature as well as extremely high quality publications available from booksellers which offer numerous ideas for Math Circle activities (see *Resources* section below).

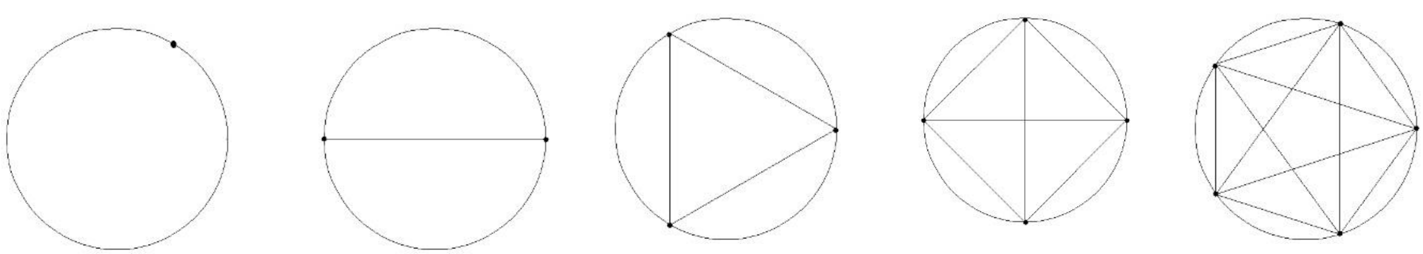
One popular Math Circle activity is the ***Regions of a Circle Problem*** (Tanton, 2010; Gowers, 2002):

Given a circle with n points on its boundary, join all pairs of these points with lines. What is the maximum number of regions formed by the lines inside the circle?

This problem can be investigated at all grade levels but is usually targeted toward high school and advanced middle school students. After posing the question, the teacher's principal responsibility is to help facilitate the discussion and exploration. How to present such topics and optimize the session is a major focus of the Math Circle literature. The implementation is dependent on the style of the teacher as well as the personalities, ages, and backgrounds of the students. In all cases, the direction of the discussion should ultimately be determined by the students. The students should be encouraged to ask questions and offer suggestions for avenues of inquiry. The teacher must also promote a balanced conversion where students are careful in forming mathematical arguments as well as offering constructive criticism to the arguments of others.

Math Circles are primarily opportunities for teachers and students to do math together. The emphasis is on the mathematical journey of the group and less on the specific results. As is the case when doing math and working on interesting problems, the group may explore wrong ideas or temporarily visit dead ends along the way. This possible frustration is part of the process which should lead to progress. The teacher's role is to promote the proper habits of mind and ultimately make sure everyone is included, engaged, and having fun.

If leading an open, mathematical discussion feels like a daunting task, there are many resources available outlining possible discussions of the topic which may prove useful. For example, James Tanton has written an outline for a Math Circle activity for this problem which can be found in the lesson plans section of the *mathcircles.org* website (2010). Tanton's activity is based on a strictly combinatorial approach to the problem. Although some gifted students may be able to have significant success with combinatorial pattern problems based solely abilities and ingenuity, the following incarnation of the problem uses basic algebra concepts. This approach was developed by the Math Circle author at Louisiana State University and starts by simply presenting the regions of the circle problem to the students, asking them to produce the answers for the n = 1, 2, 3, 4, 5.



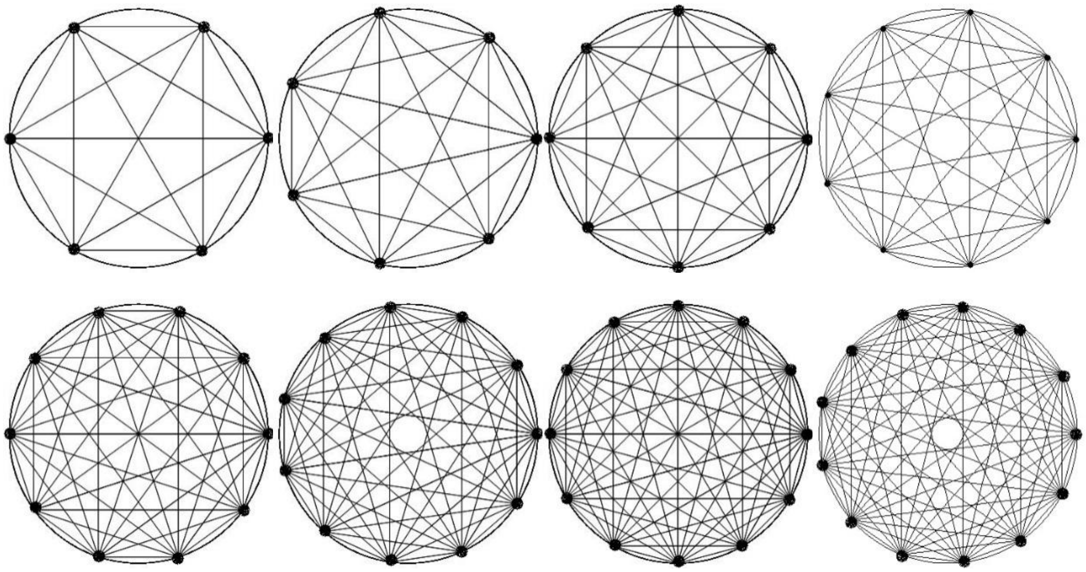
This leads to the first surprise. A student will likely propose that for any n, the number of regions created by the lines joining n pairs of dots will be 2^{n-1}. Could this conjecture be correct? With a carefully drawn circle, the students discover the initial conjecture fails for $n = 6$. The number of regions is 31 (and not 32 as the initial conjecture states). One can now encourage the students to develop further arguments to critique the initial conjecture. As n gets larger, perhaps 2^{n-1} grows too fast to represent the number of regions. This argument can be investigated pictorially. As argued in (Gowers, 2002), for $n = 30$ the 2^{n-1} hypothesis would estimate over five hundred million different regions. One could imagine (or actually make) a circle with a diameter of ten meters in a field and then place thirty pegs around the diameter, connecting all pairs of pegs with strings. As tedious as it may be to count the regions created by the intersection of these strings, it could be done. If the hypothesis were true, then there would have to be an average of six hundred regions per square centimeter, a number which seems to be much too large. Of course, this argument is by no means mathematically sound, and the line of reasoning is only mentioned to encourage students to reason practically, as well as abstractly, when they are critiquing and creating mathematical arguments.

Continuing the activity, the students compute as many terms of the sequence as possible in search of a closed form formula for the n-th term. An initial approach is to use technology. Programs such as Mathematica can easily be used to model the problem. The following is an example of such a code for $n = 9$:

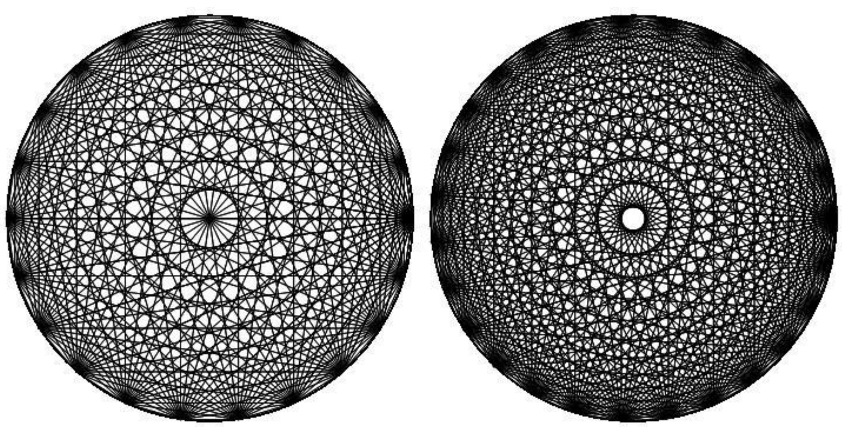
```
n=9
Graphics[Table[{Circle[{0,0},1],Line[{{Cos[i*2*Pi/n],Sin[i*2*Pi/n]},{Cos[k*2*Pi/n],Sin[k*2*Pi/n]}}]},{i,0,n},
{k,0,n}]]
```

When students begin to write such a program, they must decide how to arrange the points on the circle. Should they be placed randomly? Should they allow for curved lines? If they choose to place the points evenly spaced on the circle, the representations are symmetric. One could argue that these representations are perhaps more aesthetically beautiful, and furthermore, attention to symmetry can help students compute higher terms more easily as well as present further avenues for inquiry and discovery.

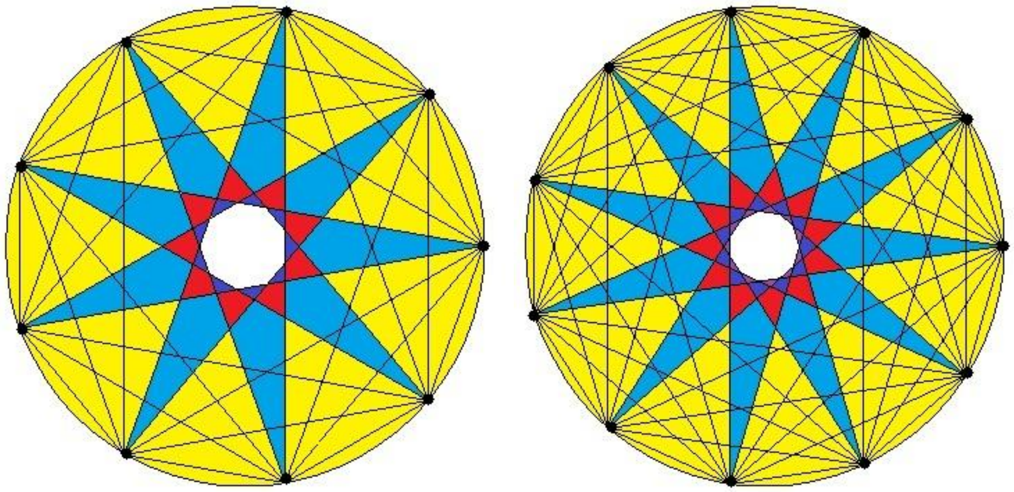
Below are representations for $n = 6, 7, \ldots, 13$ created with the given Mathematica code:



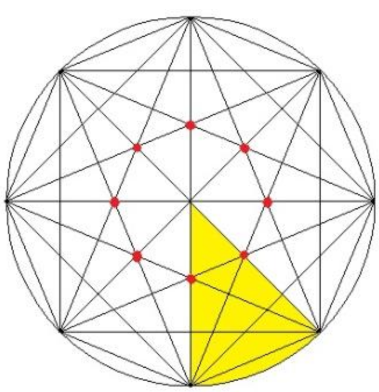
From these representations, many questions and problems follow: Can one prove for which values of n there is an intersection of lines occurring at the center of the circle? For which values is there no intersection point in the center of the circle? What happens if the program draws more than two lines intersecting at the same point, and how does this affect the number of regions? If n is odd, are there ever more than two lines intersecting at the same point? These types of questions can lead to good discussions and prepare the students for calculating higher order terms in the sequence. The representations for a large n, such as $n = 22$ and $n = 27$, could prove to be interesting math art for a classroom or bedroom wall.



After producing, digesting, and discussing the representations, the students are now ready to use the symmetries to more easily count the number of regions for higher n. For example, consider $n = 9$ and $n = 11$ shown below. From the Mathematica program's representation (with regions colored using Microsoft Paint), one can see that there are 9 yellow pie pieces (each with 11 regions), 9 light blue quadrilaterals (each with 5 regions), 9 red triangles, 9 dark blue triangles, and 1 white center region, which adds to a total of 163 regions for $n = 9$.



For $n = 11$, there are 11 yellow pie pieces (each with 23 regions), 11 light blue quadrilaterals (each with 9 regions), 11 red triangles, 11 dark blue triangles, 11 purple triangles, and 1 white center region, which is a sum total of 386 regions for $n = 11$. Further questions could be asked about the sequence generated from counting the number of regions in only the yellow pie piece shapes (or only the blue quadrilateral shapes). Can the students predict how this sequence will grow as n increases?



For even n, one must be careful with the intersection points of more than two lines. For $n = 8$, as depicted in the figure left, there are 8 yellow pie pieces each with 11 regions. There are also 8 points where three-lines intersect and 1 point where four-lines intersect. The students should be able to argue that there will be 1 additional region created by each of the three-line intersection points and 3 additional regions created by the four-line intersection point. Thus for $n = 8$, there are $8(11) + 8 + 3 = 99$ regions. What about an intersection point of five lines or six lines or *n* lines? The students should attempt to argue, in general, that any *n*-line intersection should account for potentially ½ (n-1) (n-2) additional regions. An understanding of how the number of regions is affected by points of intersections is crucial to counting the number of regions in the representations for higher *n*.

The students should now be able to come to a consensus on the number of regions for all *n* up to 12 and thus generate the following sequence:

$$a = 1, 2, 4, 8, 16, 31, 57, 99, 163, 256, 386, 562 \ldots.$$

In addition, the students should have (hopefully) developed a personal interest in the sequence and should be open to learn about different methods to more efficiently calculate higher order terms of this and other sequences. One could now direct the students to investigate the behavior of the sequence d^1 of the changes (differences) of the terms of *a*. This can serve as a nice philosophical lesson for the students: in working toward an understanding of the true nature of something, it is often helpful to understand how that something changes. Consider the sequence of first differences:

$$d^1 = 2\text{-}1, 4\text{-}2, 8\text{-}4, 16\text{-}8, 31\text{-}16, 57\text{-}31, 99\text{-}57, \ldots = 1, 2, 4, 8, 15, 26, 42, 64, 93, 130, 176, \ldots$$

Nothing should be too striking after investigating d^1, but encourage the further investigation of d^2 (the differences between consecutive terms of d^1), d^3 (the differences in d^2), and d^n (the differences in d^{n-1}), for $n > 3$. Now the students should witness the perhaps surprising revelation that by taking the fourth difference of the regions of a circle sequence *a*, they are left with the constant 1-sequence (i.e. 1,1,1,…). Of course at this stage, one cannot be sure that this is true for all terms of *a*, but it certainly holds for the first few terms. Encourage the students to move forward with the assumption that the fourth differences of *a* are constant for all terms, and in doing so, always emphasize that this assumption has to be eventually verified in order to fully understand the problem. The students should now be able to give insight to the type of equation a general formula should be.

In most high school geometry classes, students learn that if the terms of a sequence can be represented by a linear function, then the first differences of the terms are constant. Also, most students have seen that, if the general formula for a given sequence is quadratic, then the second difference of the terms must be constant. Ask the students to make a conjecture about what type of equation the general formula of a sequence should be if the p^{th} difference of that sequence is constant. A reasonable conjecture follows:

Given a sequence f whose general formula f(n) for the n^{th} term f is a polynomial of degree p, then the p^{th} difference of the terms of f must be constant.

With the above conjecture (which could be a theorem by this point if the students show interest in proving it), the students should now have a candidate for the type of equation that the general formula for *a* could be. This equation could be a fourth degree polynomial, thus one can assume

$$f(n) = c_1 n^4 + c_2 n^3 + c_3 n^2 + c_4 n + c_5.$$

Using the first four terms of *a*, the following system of equations follows:

$$\begin{aligned}
a(1) &= c_1 + c_2 + c_3 + c_4 + c_5 &&= 1\\
a(2) &= 16\,c_1 + 8\,c_2 + 4\,c_3 + 2\,c_4 + c_5 &&= 2\\
a(3) &= 81\,c_1 + 27\,c_2 + 9\,c_3 + 3\,c_4 + c_5 &&= 4\\
a(4) &= 256\,c_1 + 64\,c_2 + 16\,c_3 + 4\,c_4 + c_5 &&= 8\\
a(5) &= 625\,c_1 + 125\,c_2 + 25\,c_3 + 5\,c_4 + c_5 &&= 16.
\end{aligned}$$

This system can be solved by hand or, perhaps more efficiently, by using technology (a TI-83 graphing calculator works nicely). The result

$$a(n) = \tfrac{1}{24}\, n^4 - \tfrac{6}{24}\, n^3 + \tfrac{23}{24}\, n^2 - \tfrac{18}{24}\, n + 1$$

is a candidate for a general formula for *a*. Remarkably, the formula does hold true for all known values of *a*, and thus the value of $a(n)$ appears to be an integer for all integer *n*. What else can the students learn about the original regions of the circle problem by looking at this formula? There is little insight that one can draw from this representation of a potential solution other than the fact that the final solution needs to be an *integer polynomial*, meaning that the formula produces integer value outputs for all integer value inputs. Perhaps there is a more insightful representation for the general formula yet to be discovered.

In searching for this representation, the students should be encouraged to research integer polynomials. The first result from a Google search informs that every integer polynomial $f(x)$ of degree p in the variable x can be written in the form

$$f(x) = c_0 + c_1 \binom{x}{1} + c_2 \binom{x}{2} + \ldots + c_p \binom{x}{p}$$

where the c_i are integers (Weisstein, 2012). The website directs readers to Nagell (1951) for the proof of the claim. The proof can be found in an online version of the text and can clearly be presented to the group or formulated independently by motivated students. This can be a great way to introduce students to mathematical induction for those who have little experience writing proofs.

It is clear that the desired formula $a(n)$ must be an integer representing polynomial. From the brief diversion into the study of integer representing polynomials, one must be able to write $a(n)$ as follows:

$$a(n) = \tfrac{1}{24} n^4 - \tfrac{6}{24} n^3 + \tfrac{23}{24} n^2 - \tfrac{18}{24} n + 1 = c_0 + c_1 \binom{x}{1} + c_2 \binom{x}{2} + c_3 \binom{x}{3} + c_4 \binom{x}{4}$$

The students can now use the first four terms of a to easily solve for the above coefficients.

$$\begin{array}{lll} a(0) = 1 = c_0 & \rightarrow & c_0 = 1 \\ a(1) = 1 = 1 + c_1 & \rightarrow & c_1 = 0 \\ a(2) = 2 = 1 + c_2 & \rightarrow & c_2 = 1 \\ a(3) = 4 = 1 + 3 + c_3 & \rightarrow & c_3 = 0 \\ a(4) = 8 = 1 + 6 + c_4 & \rightarrow & c_4 = 1 \end{array}$$

With this new representation

$$a(n) = 1 + \binom{n}{2} + \binom{n}{4},$$

the students are ready to return to the regions of the circle problem's pictorial representation. Does the new representation still accurately represent the terms of a? How does this new representation differ from the previous representation

$$a(n) = \tfrac{1}{24} n^4 - \tfrac{6}{24} n^3 + \tfrac{23}{24} n^2 - \tfrac{18}{24} n + 1?$$

Most importantly, are the students able to draw insight to the nature of the problem from the new representation? Looking at a circle with n points at its boundary, every two points determine a line and every four points determine an intersection point of two lines. Further consideration shows that there are $\binom{n}{2}$ lines and

$\binom{n}{4}$ points of intersection. The new representation for $a(n)$ should illustrate to the students that the maximal number of regions in the circle produced by connecting n points on the boundary with lines should be identical to 1 plus the number of lines plus the number of points of intersection of those lines; hence

$$a(n) = 1 + L_n + I_n$$

where $L_n = \binom{n}{2}$ (the number of lines) and $I_n = \binom{n}{4}$ (the number of intersections). Finally, the students have a conjecture that they should be ready to prove.

> *In any circle with n points on the boundary, the maximal number a(n) of different regions that can be generated by connecting the n points with lines is given by* $a(n)= 1 + L_n + I_n$ *where* L_n *is the number of lines that can be drawn and* I_n *is the number of intersections that can be obtained.*

This proof can be found in Tanton (2010). Now the students have proved that the general formula for the sequence a is given by $a(n) = 1 + \binom{n}{2} + \binom{n}{4}$. At this point the session can be extended using Tanton (2010) to explore other combinatorial and graph theoretic generalizations of the result. Alternatively, the session could be extended by having the students explore integer representing polynomials. What do the graphs of these integer polynomials look like? How does one create a sequence that coincides with the powers of 2 up to 2^n for any n? If the students enjoy working with sequences, then one also has the option to introduce the z-transform which serves as a useful tool for the study of all sequences. Another session could return to the regions of the circle sequence a and use the z-transform to find a general formula for $a(n)$.

Multiple directions for exploration and generalization are typical to the *Regions of a Circle Problem* and many other Math Circle activities. The above details just one possible path for a circle focused on this topic. Regardless of the topic, teachers must develop their own personal ways of making students successful in asking questions and constructing arguments to make progress on mathematical tasks to demonstrate their proficiency with the *Standards of Mathematical Practice*. Math Circle activities are ideal, fun settings for students and teachers to engage in mathematics so all participants foster the proper mathematical habits of mind. All teachers should take advantage of the wonderful resources that the Math Circle movement has to offer.

Resources:

For information on the PARCC and SMARTER Balanced Assessment see:

- http://balancedassessment.concord.org/
- http://web.me.com/acaciatc/UACC/PARCC_Resources.html

For Math Circle Activities and Lessons plans see:

- National Association of Math Circles website: http://www.mathcircles.org/
- Math Teachers' Circle Network website: http://www.mathteacherscircle.org/
- Vandervelde, S. (2009). *Circle in a Box*. Berkeley, California: Mathematical Sciences Research Institute.
- Stankova, Z., & Rike, T. (Eds). (2008). *A Decade of the Berkeley Math Circle*. Berkeley, California: Mathematical Sciences Research Institute.
- Fomin, D., Genkin, S., & Itenber, I. (1996) *Mathematical Circles (Russian Experience).* Providence, Rhode Island: American Mathematical Society.
- The Math Circle website: http://www.themathcircle.org/

Currently there are three Louisiana Math Circles. Anyone interested in participating or needing guidance in starting their own circles should contact a group leader.

Teachers Circles:

- **Acadiana Math Teachers Circle** meets once a month from September until April at the University of Louisiana, Lafayette to work on problem sets under the guidance of ULL mathematicians. The group discusses a variety of approaches to problems and new ideas for increasing enthusiasm in the classroom and developing their students' critical thinking skills. Please contact Cat McKay (cmckay7930@earthlink.net) for more information.
  Website: http://www.ucs.louisiana.edu/~cxe2945/teachercircle.html
- The **North Louisiana Math Teachers Circle** is a new initiative that empowers middle school math teachers to bring new excitement and interest in mathematics to their students. This circle meets on Monday evenings during the academic year and is based out of Louisiana State University - Shreveport. Please contact Judith Covington (Judith.Covington@lsus.edu) for more information.
  Press Release: http://www.ktbs.com/news/North-LA-Math-Teachers-Circle/-/144844/9270758/-/hd6w1pz/-/index.html

Circles for Students:

- The **LSU MathCircle Summer Enrichment Program** is a three-week summer program at LSU for motivated and math-loving high school students. The program focuses on exposing students to subjects not usually taught at a high school level. For more information, please visit our website.
  Website: http://www.cain.lsu.edu/MathCircle/about.html

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Lee Windsperger *is a doctoral student in the Department of Mathematics at Louisiana State University – Baton Rouge. In addition to mathematical research, he has been involved in a variety of mathematical outreach and education programs including enrichment, college readiness, and professional development for practicing teachers programs. He is also a coordinator for the LSU MathCircle, a residential summer math enrichment program for gifted high school students.*

Guest Column

Insight from an NCTM Board Member – Latrenda Knighten

On the surface, November 2, 2010, began and *almost* ended like many other days in my busy life. After a long, but rewarding day at school, I came home and performed a routine task that like many of you I perform each evening – reading and responding to email. The subject line of one email, *NCTM Election Results*, immediately caught my eye. The email was from Kichoon Yang, Executive Director of the National Council of Teachers of Mathematics (NCTM), informing me that I had been elected to the NCTM Board of Directors for a three-year term! In April 2011 at the NCTM Annual Meeting, along with four other members of the "freshmen" class, I officially became a member of the NCTM Board of Directors.

If anyone had asked me at the beginning of my teaching career 20+ years ago, 10 years ago or even five years ago if I had ever imagined myself serving in this capacity – my response would have been "no way." My journey to becoming a member of the NCTM Board of Directors involves a long, sometimes winding, fun-filled, personally and professionally fulfilling path. I can't pinpoint the actual beginning of my journey; however, joining my local and state affiliate groups early in my teaching career are events that can be found at the beginning of my journey. Membership in these organizations provided me with the opportunity to network with fellow mathematics educators. Like many LATM (Louisiana Association of Teachers of Mathematics) members, I quickly learned the benefits of being able to obtain first-hand knowledge of mathematics initiatives and reforms and networking with a diverse group of mathematics educators as a member of an NCTM affiliate group. Over time I've had the opportunity to serve as a volunteer in my local affiliate group (BRACTM – Baton Rouge Area Council of Teachers of Mathematics) and LATM in many capacities such as conference volunteer, presenter at conferences, and fulfilling various leadership roles. The most important lesson I've learned from my varied experiences is the importance of getting involved. I'm reminded of the quotation made famous by John F. Kennedy – "Ask not what your country can do for you, ask what you can do for your country." As an

NCTM member, you may ask, “What can I do for NCTM?” One of the most important things we as members can do for NCTM is to get involved at all levels. NCTM like LATM and other affiliates is a volunteer driven organization. I’ve listed a few suggestions below of things members can do at all levels to support NCTM and get involved.

1. One of the first steps to getting involved is to join and maintain your NCTM membership. NCTM serves as the premier professional organization for mathematics educators in the United States, Canada and other international arenas. Supporting NCTM by being a member helps the Council continue to play a leading role in mathematics reform and provide a wide array of professional development offerings (print, online, and face to face) for mathematics teachers. Consider an E-membership as a low-cost option which still provides online access to member benefits and online access to the NCTM journal of your choice.
2. Support your local and state affiliate (LATM) by maintaining your membership and serving as a volunteer for events sponsored by these groups. LATM and local groups are affiliates of NCTM with similar goals and activities as NCTM with one of those activities being an annual conference. Support your local AG and LATM by volunteering to serve as a presenter for conferences and other professional development offerings. Membership in local and state affiliates allows members to support mathematics education reform, network with others, and serve in leadership roles. Affiliate leaders and members have the opportunity to attend NCTM-sponsored events such as the Affiliate Leaders Conference and in some instances host NCTM regional conferences and annual meetings such as the recent 2010 New Orleans Regional Conference and the upcoming 2014 NCTM Annual Meeting. NCTM regional conferences and annual meetings provide members with many opportunities to serve as volunteers in many capacities (committee member, conference volunteer, presenter, etc.)
3. Planning to attend a regional conference or annual meeting? NCTM issues a call for volunteers to all conference attendees for all regional conferences and annual meetings. Most volunteer shifts require a

one-hour or two-hour commitment and provide a wonderful opportunity to network with a diverse group of mathematics educators, support and provide a much needed service to NCTM, be recognized by name on the NCTM website and in the conference update newsletter, and in some instances receive a complimentary “conference volunteer” shirt.

4. Use and promote the use of NCTM journals, publications, and online resources such as the lessons and activities available on Illuminations. As educators, we’re all aware that good news travels well. NCTM has a host of award-winning resources available to support mathematics educators at all levels (K-12 educators, teacher educators and researchers, etc.). Many of these resources are some of the best kept secrets available to educators. Help spread the word and share the secrets of NCTM resources with colleagues, parents, and pre-service educators.
5. Volunteer to present a session or workshop at an NCTM regional conference or annual meeting. Serving as a presenter at an NCTM conference allows presenters to share teaching ideas and practices with colleagues while networking with peers. Interested in speaking at an upcoming 2013 regional conference? Speaker proposal forms for the 2013 regional conferences must be submitted by September 30, 2012. Visit the NCTM web page and select the conferences link to access the online speaker proposal forms.
6. Write for NCTM. Interested in sharing innovative teaching ideas, sharing ideas for linking research to practice, or proposing an idea for a new book? Consider submitting a proposal to NCTM. NCTM has several award-winning journals for K-12 mathematics educators and teacher educators at all levels to support budding writers in their endeavors to publish and share ideas with peers. For those writers interested in submitting lesson activities for students in grades 5 – 12, consider submitting a lesson activity to Students Explorations in Mathematics. Writers interested in writing a book or proposing a book idea to NCTM can submit their ideas to the NCTM Director of Publications. Visit the NCTM web page for guidelines and submission requirements.

7. One last suggestion for getting involved is to apply to serve on an NCTM committee. NCTM has twelve regular committees such as the Editorial Panels for each of the journals, the Research Committee, Professional Services Development Committee, and the Affiliate Services Committees to name a few. These committees provide a variety of services to the membership and are designed to appeal to the diverse membership. Visit the NCTM web page to learn more about NCTM committees and their duties.

These are just a few suggestions to help members get started on a long path of volunteerism in NCTM. While these suggestions appeal to a wide variety of members with diverse talents and interests, there are many other ways to get involved with NCTM. The most important thing to remember is to GET INVOLVED!

Latrenda Knighten *(LDKnighten@aol.com) is currently an elementary mathematics coach at Melrose Elementary in Baton Rouge. She has more than 23 years experience as an educator where she has served as a classroom teacher, elementary mathematics specialist, elementary science specialist, and instructional coach. Latrenda has also worked at the state level with the Louisiana Department of Education in assessment development for the mathematics LEAP assessments. She has served on several NCTM committees and held several leadership positions in BRACTM and LATM including serving as president of both BRACTM and LATM. She presently serves on the BRACTM Executive Council as NCTM Representative. Latrenda can be reached at LDKnighten@aol.com.*

Preservice Point of View: Multiplying a Two-Digit Number by a Single-Digit Number

Nell McAnelly and DesLey Plaisance

This section of the *LATM Journal* is designed to link teachers and future teachers. In each journal, responses to a mathematical task by preservice teachers are presented. It is anticipated that these responses will provide insight into understanding, reveal possible misconceptions, and suggest implications for improved instruction. In addition, it is expected that this section will initiate a dialogue on concept development that will better prepare future teachers and reinforce the practices of current teachers.

Preservice elementary education students enrolled in a senior-level mathematics course specifically designed for elementary education majors were given the following problem:

Consider the following problem:

23 x 4 or

$$\begin{array}{r} 23 \\ \underline{x\ \ 4} \end{array}$$

First, find the product and explain in words how you found that product. You may use your "work" and diagrams along with words to explain the process for finding the product. Please be clear so that the reader can follow your reasoning.

Second, describe you would explain to a 5th grade student how to find the product. Again, you may use words, "math work," and/or diagrams within this description. Please be clear so that the reader can follow your reasoning.

The students were presented this problem approximately two months into the semester without any directions other than those given within the problem. For the purpose of this specific journal column, the responses to the first question will be discussed.

As students think about this problem and how it can be solved, they may not think that this problem is difficult. Some may even consider solving the problem mentally. Students were not told to avoid mental math, but they had to "explain in words" how they found the product.

Consider how you would solve this problem. What method(s) might you, the reader, utilize in solving the problem? Jot down your thoughts about solving the problem before reading further.

In considering this problem for the column, we wanted to explore the various methods employed by these preservice teachers in solving the problems themselves. We also thought about the obvious ways that the students might solve this problem. One of the obvious ways might be the standard algorithm for multiplication. In many instances when the standard algorithm is taught in the classroom, place value is not even mentioned. An example of this method being taught (without mentioning place value) can be found in a YouTube video titled "Multiplying by One Digit Number." The video can be viewed by visiting the following link: http://www.youtube.com/watch?v=dk65P77hTCQ&feature=related

Another way to solve the problem would be to use mental math utilizing the distributive property – basically 4(3 + 20) or 4(20 + 3). Mentally one might think about "four times three" which is twelve. Then, one might think about "four times twenty" which is eighty. Those are usually referred to as the "partial products." Once the partial products are found, then those are added mentally: 12 + 80 = 92. The product of 23 and 4 is 92.

An alternate written method that is parallel to the mental math method described is using the partial products in a format similar to the way the traditional algorithm is written. However, there is emphasis on place value that is not found in the traditional algorithm. An example of this method can be found in another YouTube video titled "New Math with a Focus on Place Value" and can be viewed at http://www.youtube.com/watch?v=_zzAdfs_Src&feature=related.

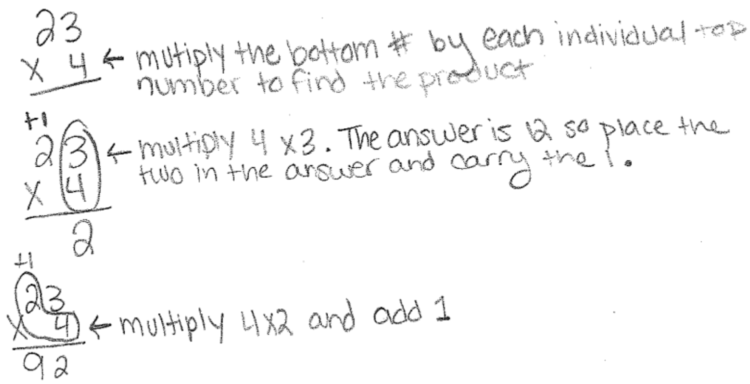
There may be other methods that are slightly different than those mentioned. Was one of your methods different from the ones described here?

Four classes of students were asked to solve this problem. All answers were reviewed and categorized into the following categories: 1) Traditional algorithm; 2) Utilizing Distributive Property; 3) Utilizing Two Methods for Explanation. The following are examples of student work from the three categories:

1) Traditional Algorithm

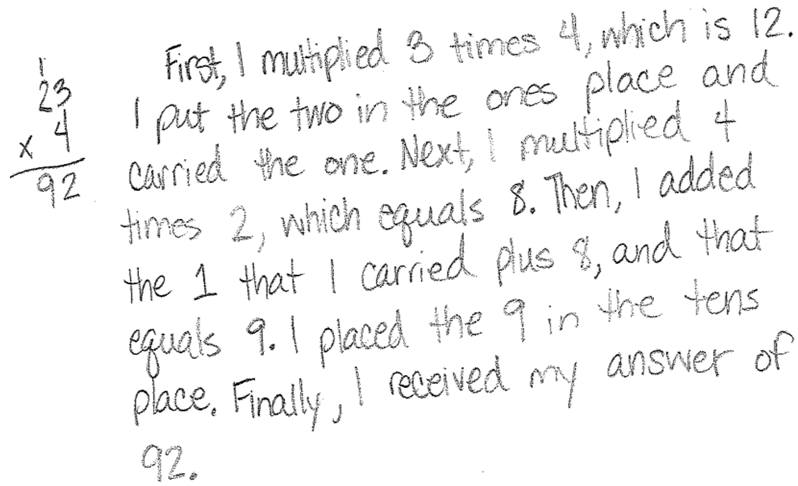
Example 1:

Student uses traditional algorithm, but does not indicate place value at any time.



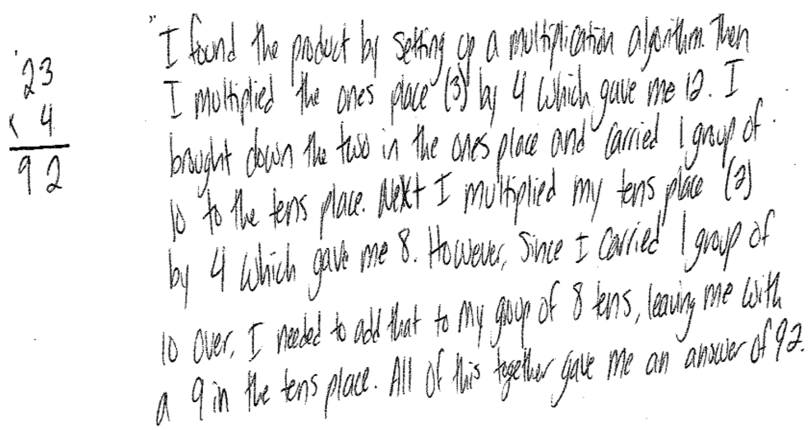
Example 2:

Student uses traditional algorithm, but does make note of place value. However, when indicating there is a "carry" indicates a carry of "one" and not a carry of "one ten."



Example 3:

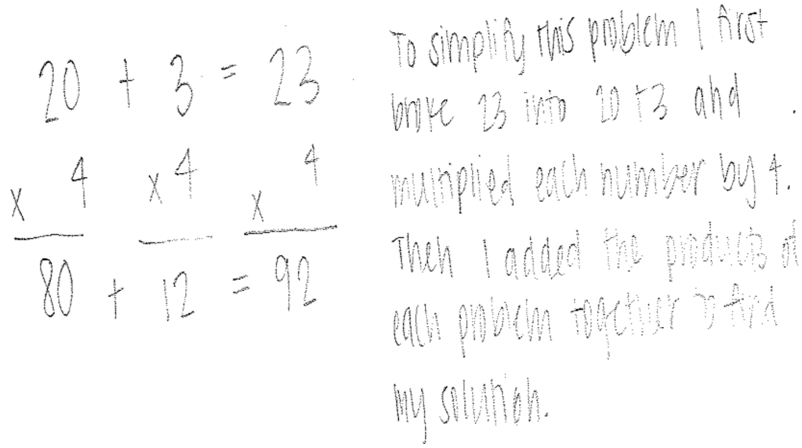
Student uses traditional algorithm and makes note of place value throughout explanation.



2) Utilizing Distributive Property

Example 1:

Student used the distributive property by decomposing 23 into 20 added to 3. The manner in which the student wrote the problem is not the “conventional” form of the distributive property, but does utilize that property.

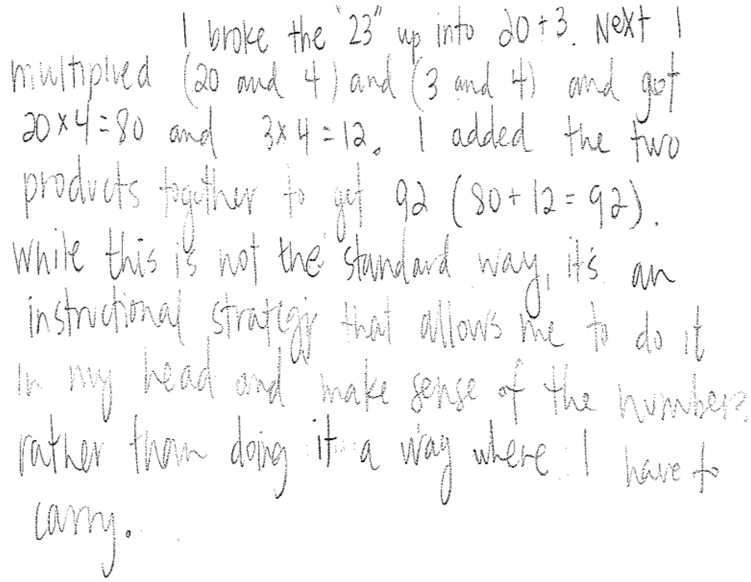


20 + 3 = 23
x 4   x 4   x 4
80 + 12 = 92

To simplify this problem I first broke 23 into 20 + 3 and multiplied each number by 4. Then I added the products of each problem together to find my solution.

Example 2:

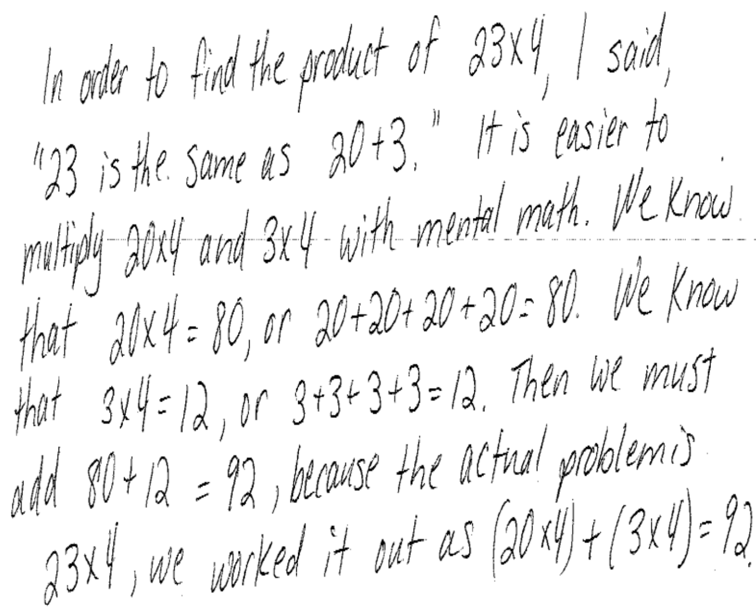
Student explained procedure along with “why” this strategy works for the student.



I broke the "23" up into 20+3. Next I multiplied (20 and 4) and (3 and 4) and got 20x4=80 and 3x4=12. I added the two products together to get 92 (80+12=92). While this is not the standard way, it's an instructional strategy that allows me to do it in my head and make sense of the numbers rather than doing it a way where I have to carry.

Example 3:

Student explained method and reinforced the multiplication by showing equivalent version of addition.

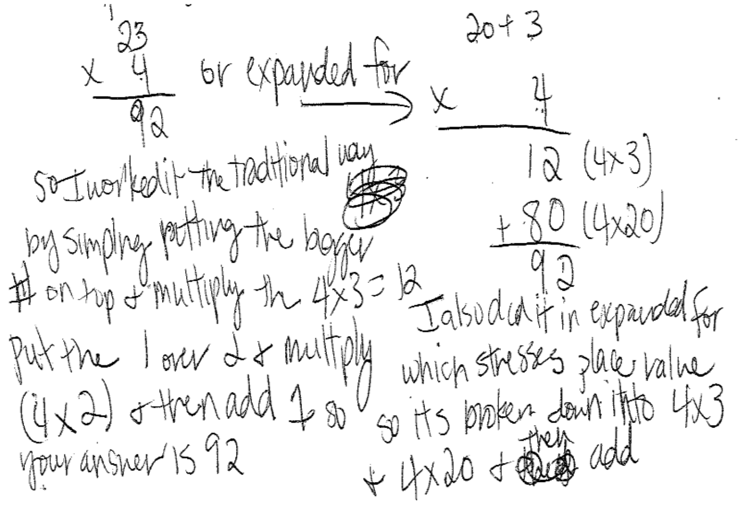


In order to find the product of 23x4, I said, "23 is the same as 20+3." It is easier to multiply 20x4 and 3x4 with mental math. We know that 20x4=80, or 20+20+20+20=80. We know that 3x4=12, or 3+3+3+3=12. Then we must add 80+12=92, because the actual problem is 23x4, we worked it out as (20x4)+(3x4)=92.

3) Utilizing Two Methods for Explanation

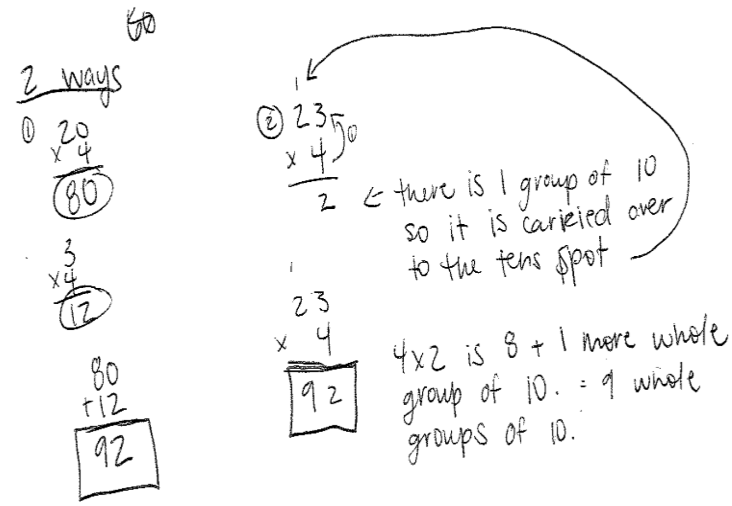
Example 1:

Student used the traditional method and a form of the distributive property. In the traditional algorithm, the student does not mention place value. While using the distributive property (referred to as expanded form), he mentions that the "expanded" form stresses place value which indicates that the student is somewhat aware of the importance of place value.



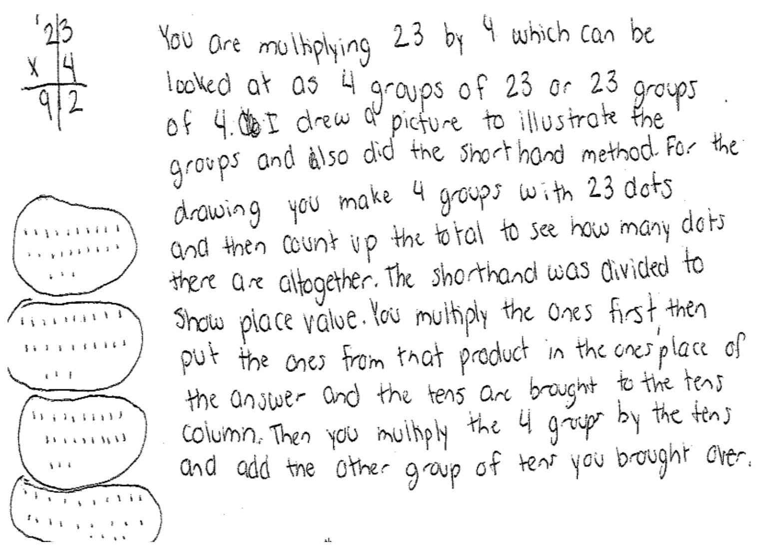
Example 2:

Student used a form of the distributive property and the traditional algorithm. In this example, the student explains "carrying" using appropriate place value language.



Example 3:

Student used the traditional algorithm and a drawing. He referred to the traditional algorithm as the "shorthand method." The drawing "matched" the problem if it were looked at as "multiplying 23 by 4" or "4 groups of 23." The student did indicate that the problem could be interpreted as "23 groups of 4."



***DesLey Plaisance** is currently an Assistant Professor in the Department of Mathematics and Computer Science at Nicholls State University and serves as Coordinator of Graduate Studies in Mathematics. She teaches undergraduate courses in mathematics for education majors and graduate courses in mathematics curriculum/research. Plaisance's primary research focus is mathematics anxiety of preservice elementary teachers. She is the LATM Journal Coordinator of the LATM Executive Council.*

***Nell McAnelly** is Co-Director of the Gordon A. Cain Center for Scientific, Mathematical, Engineering, and Technology Literacy and is a mathematics instructor at LSU-BR. Her most recent teaching focuses on mathematics content courses for elementary education majors. McAnelly oversees numerous professional development projects for mathematics and science teachers from kindergarten through twelfth grade and is the Executive Director of the Quality Science and Math Grant Program. She presently serves on the LATM Executive Council as Treasurer.*

Tech Talk: Save it to "the Cloud" or How to Leave Your Jump Drive at Home

Lori C. Soule

When was the last time you felt like one of your students because you forgot your jump drive at home and the only copy of today's exam is stored on the drive? Well, fret no more. There is an easy and free solution to forgetting your jump drive. As long as you have an Internet connection, you will have access to your files.

There are several "cloud based" file sharing services that allow you to access and share your files from anywhere. These services will also work great for use as extended storage on your mobile devices, such as an iPad. For this Tech Talk discussion, I will be taking a look at Box.net, SugarSync, and Dropbox.

The first "cloud" we will stop at is Box.net. Box.net (www.box.com) is easy to setup. At the website, you sign up for an account and there is a tour of the site showing you how to use the site's features. There is no client desktop software available for Box; you will need to access the website to upload files. As a personal user, you get 5MB free and you can organize all your files into folders. Additional storage can be purchased for a fee.

Box allows you to share files, such as videos and presentations, which are too large for emails; this is accomplished by sending a link to the file via email or text message. An entire folder of files can be shared which makes working on a group project so much easier and you will receive real-time updates letting you know when someone has viewed, edited, or commented on your file.

Box can be accessed via mobile devices. These mobile devices include (1) Android phones and tablets, (2) Apple iPods, iPhones, and iPads, and (3) BlackBerry smart phone and PlayBook. When using a mobile device, you can view the content of a file, or share a file, or even folders, with others.

The next "cloud" we will be visiting is SugarSync. You can try SugarSync (www.sugarsync.com) for free for 30 days or start with a free 5GB account. SugarSync will allow you to download and install client software on your computer during the account sign up. You will be prompted for a name and icon for your computer and then you can choose folders to back up on the SugarSync's servers and make available to other computers/devices including those using other operating systems. The installer creates and places a "Magic Briefcase" folder on your desktop. Files can be dropped into the Magic Briefcase and these files will be synchronized across your other computers/devices. In addition, the installer will place a SugarSync icon in your system tray where you can change your synchronization options and manage your backed-up files.

You can increase your SugarSync storage for free by referring friends. For every friend you refer, both will receive 500MB of bonus space to a maximum of 32GB. If your friend joins a paid plan, both you and your friend will get 10GB of free storage with no limit in maximum bonus space you can earn.

SugarSync will continuously backup the selected documents, photos, videos, and music from all your computers/devices to your secure personal cloud. The previous five versions of each file are stored just in case you need to reference or recover them in the future. With SugarSync, you can send any file of any size to anyone; you can generate a public link and share files directly through Facebook, Twitter, email, IM, or your blog. In addition, you can stream music or show photos from any of your computers directly from the SugarSync website or on your mobile device using the SugarSync app. If you are concerned about security, all data transfers are encrypted and data storage is secure and redundant.

SugarSync can be used on a broad set of computing and mobile platforms. SugarSync supports:

DESKTOP	**WEB**	**MOBILE**
Windows 7	Internet Explorer 7.0+	iPad
Windows Vista	Chrome 8+	iPhone
Windows XP	Firefox 3.6+	BlackBerry
Mac OSX	Safari 5.0+	Windows Mobile
		Android
		Symbian
		SugarSync Mobile Web

The final "cloud" to be discussed is Dropbox. Dropbox (www.dropbox.com) is similar to SugarSync in many ways. You can download and install client software on your computer. The installer will place a Dropbox icon in your system tray where you can change your preferences, pause your syncing, access your files, and get more space. Instead of a "Magic Briefcase" placed on your desktop, a "My Dropbox" folder is created within your My Documents folder. As a result of the folder creation and placement, the learning curve to use Dropbox is minimal. A distinct difference is the amount of free storage available. A free Dropbox account is 2GB in size. Like SugarSync, you can increase your free space through referrals. For every friend who joins and installs Dropbox, you both will receive 250MB of bonus space up to the limit of 8GB.

Dropbox allows you to invite others to a folder; it will be as if you saved the folder on their computer. You can send a link to any file in your Public folder. Dropbox transfers just the parts of a file that changes, not the entire file. One month's history of your work is saved, any changes can be undone, and files can be deleted. Dropbox works with Windows, Mac, Linux, iPad, iPhone, Android, and BlackBerry.

I have accounts with all three of these cloud services. I have been using Dropbox for almost two years. It was one of the first cloud services, besides Google Docs, that was available. Recently, I opened a Box account and a SugarSync account. I decided to open the Box account because of the amount of free storage, 50GB, they were offering for a limited time; fifty gigabytes is more than I will need for the near future. I wanted to compare SugarSync to Dropbox so that was my reasoning for opening SugarSync account. Now that I have all three, I use Dropbox the most. I guess I'm a creature of habit and some habits are hard to break!

Lori C. Soule *is the Director for Advancing Faculty Engagement (CAFÉ) at Nicholls State University and an Assistant Professor of Interdisciplinary Studies. Dr. Soule received her PhD from Louisiana State University – Baton Rouge in Educational Technology. She can be reached at lori.soule@nicholls.edu.*